程式45　Stiffness and flexibility

1. For a rod, we have stiffness and flexibility matrices,

$$[K]=\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix},\ [F]=\frac{1}{4}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix},$$

For a beam, we have stiffness and flexibility matrices,

$$[K]=\begin{bmatrix}12 & 6 & -12 & 6\\ 6 & 4 & -6 & 2\\ -12 & -6 & 12 & -6\\ 6 & 2 & -6 & 4\end{bmatrix},\ [F]=\frac{1}{150}\begin{bmatrix}2 & 1 & -2 & 1\\ 1 & 38 & -1 & -37\\ -2 & -1 & 2 & -1\\ 1 & -37 & -1 & 38\end{bmatrix},$$

We have employed pseudo-inverse or truncated SVD technique to determine the inverse of a singular matrix.　Now we will propose another method.

$$U\tilde{t}=T\tilde{u},$$

T^{-1} does not exist.

$$T=C+\Phi_r\Psi_r^T=\begin{bmatrix}\Phi_\ell & \Phi_r\end{bmatrix}\left[\begin{array}{c|c}\Sigma_\ell & 0\\ \hline 0 & \Sigma_r\end{array}\right]\begin{bmatrix}\Psi_\ell^T\\ \Psi_r^T\end{bmatrix},$$

$$T\Psi_r=0,$$

$$\tilde{u}=\tilde{u}_c+\tilde{u}_p=\Psi_r\tilde{y}+\tilde{u}_p,$$

$$\tilde{u}_p\cdot\tilde{\psi}=0\quad\Rightarrow\quad\Phi_r^T\tilde{u}_p=0,$$

$$\Rightarrow U\tilde{t}=T\tilde{u}=T\left(\Psi_r\tilde{y}+\tilde{u}_p\right)=\left(C+\Phi_r\Psi_r^T\right)\tilde{u}_p=C\tilde{u}_p,\ u_p=C^{-1}U\tilde{t}.$$

2. Solve the Laplace problem with the following boundary conditions.

(a) $u(0)=100,\ t(0)=0$,

(b) $u(0)=100,\ t(1)=0$,

(c) $u(0)=100,\ t(1)=100$.