程式 95 Elliptic Cylinder with Two Cuts Under Torsion

Governing equation： $\nabla^2 u = 0$

Calculate the torsional rigidity C numerically for the cases where the ratio of the semiaxes is 1/4,1/2 and 3/4

Analytical solution：

The torsion function is

$$u(a,b) = -c^2 \sinh^2 a \sin^2 b + \frac{8b^2}{p}\sum_{n=0}^{\infty}\frac{\cosh(2n+1)a}{\cosh(2n+1)a_0}\frac{\sin(2n+1)b}{(1-4n^2)(2n+3)}.$$

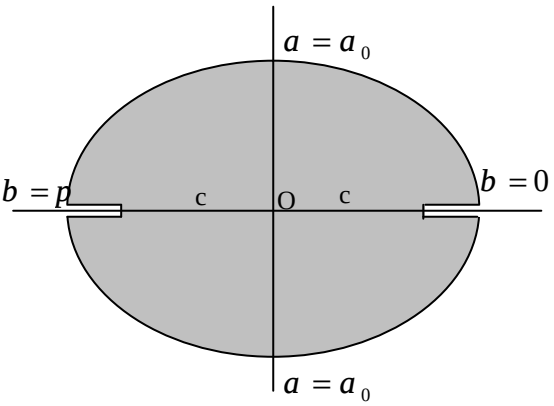
The torsional rigidity is

$$\frac{C}{C_0} = \frac{1+(b^2/a^2)}{b/a}\left[\frac{32}{p^2}(1-\frac{b^2}{a^2})S - \frac{b}{2a}\right],$$

$$S = \sum_{n=1}^{\infty}\left[\frac{(2n-1)\sinh(2n+3)a_0 + (2n+3)\sinh(2n-1)a_0}{2(2n+1)}\right.$$

$$\left. - \sinh(2n+1)a_0\right]\frac{1}{(1-4n^2)(2n-1)(2n+3)^2\cosh(2n+1)a_0}$$

where $\tanh a_0 = b/a$, and C_0 is the torsional rigidity of the elliptic without the cuts.



Numerical result: $\left.\frac{C}{C_0}\right|_{b/a=1/4} = 0.997, \left.\frac{C}{C_0}\right|_{b/a=1/2} = 0.970, \left.\frac{C}{C_0}\right|_{b/a=3/4} = 0.826$

Reference:

1. N.N. Lebedev, I.P. Skalskaya, T.S. Uflyand, Work Problem in Applied Mathematics, Dover, New York, 1979.