In the course, we have derived the constraint among u (0), u (l), t (0) and t (l) by using the integral equation for the domain piont approaching boundary ($s = 0^+$ and l^-) for UT and LM formulations.

Please derive the constraint by the null-field integral equation approaching boundary ($s = 0^-$ and l^+) for UT and LM formulations.

Sol:

$$U(s, x) = \frac{1}{2} |x - s| = \begin{cases} \frac{1}{2}(x - s), & x > s \\ \frac{1}{2}(s - x), & s > x \end{cases}$$

$$T(s, x) = \frac{\partial U(s, x)}{\partial s} = \begin{cases} \frac{-1}{2}, & x > s \\ \frac{1}{2}, & s > x \end{cases}$$

$$0 = \left[T(s, x)\, u(s) - \frac{du(s)}{ds} U(s, x)\right] \Big|_{s=0}^{s=l} \quad (x < 0,\ x > l)$$

(a) $x = 0^-$

$$0 = [T(s, 0^-)\, u(s) - t(s)\, U(s, 0^-)] |_{s=0}^{s=l}$$

$$0 = T(l, 0^-)\, u(l) - t(l)\, U(l, 0^-) - T(0, 0^-)\, u(0) + t(0)\, U(0, 0^-)$$

$$0 = \frac{1}{2} b - \frac{l}{2} u'(l) - \frac{1}{2} a$$

$$t(l) = \frac{1}{l}(b - a)$$

(b) $x = l^+$

$$0 = [T(s, l^+)\, u(s) - t(s)\, U(s, l^+)] |_{x=0}^{x=l} \quad (x < 0,\ x > l)$$

$$0 = T(l, l^+)\, u(l) - t(l)\, U(l, l^+) - T(0, l^+)\, u(0) + t(0)\, U(0, l^+)$$

$$0 = \frac{-1}{2} b + \frac{1}{2} a + \frac{l}{2} u'(0)$$

$$u'(0) = \frac{1}{l}(b - a)$$

$$L(s, x) = \frac{\partial U(s, x)}{\partial x} = \begin{cases} \frac{1}{2}, & x > s \\ \frac{-1}{2}, & s > x \end{cases}$$

$$M(s, x) = \frac{\partial L(s, x)}{\partial s} = \begin{cases} 0, & x > s \\ 0, & s > x \end{cases}$$

$$0 = \left[\frac{\partial T(s, x)}{\partial x} u(s) - \frac{\partial U(s, x)}{\partial x} t(s)\right] \Big|_{s=0}^{s=l} \quad (x < 0,\ x > l)$$

$$0 = [M(s, x)\, u(s) - L(s, x)\, t(s)] |_{s=0}^{s=l} \quad (x < 0,\ x > l)$$

(c) $x = 0^-$

$$0 = [M(s, 0^-)\, u(s) - L(s, 0^-)\, t(s)] |_{s=0}^{s=l}$$

$$0 = M(l, 0^-)\, u(l) - L(l, 0^-)\, t(l) - M(0, 0^-)\, u(0) + L(0, 0^-)\, t(0)$$

$$0 = \frac{1}{2} t(l) - \frac{1}{2} t(0) \text{ -------- } (1)$$

(d) $x = l^+$

$$0 = [M(s, l^+)\, u(s) - L(s, l^+)\, t(s)] |_{s=0}^{s=l}$$

$$0 = M(l, l^+)\, u(l) - L(l, l^+)\, t(l) - M(0, l^+)\, u(0) + L(0, l^+)\, t(0)$$

$$0 = -\frac{1}{2} t(l) + \frac{1}{2} t(0) \text{ -------- } (2)$$

由式子 (1) (2) 無法求出t (1), t (0)

參考解答 by 周克勳