Table Addition theorem or Degenerate kernel

1-D
$e^{x-s}=\dfrac{e^x}{e^s}$
$\sin(x-s)=\sin x\cos s-\cos x\sin s$
$\cos(x-s)=\cos x\cos s+\sin x\sin s$
$\dfrac{1}{2}\lvert x-s\rvert=\begin{cases}\dfrac{1}{2}x-\dfrac{1}{2}s, & x>s\\ \dfrac{1}{2}s-\dfrac{1}{2}x, & s>x\end{cases}$
2-D
$U(x,s)=\ln r=\begin{cases}\ln R-\sum_{m=1}^{\infty}\dfrac{1}{m}\left(\dfrac{\rho}{R}\right)^m\cos m(\theta-\phi), & R>\rho\\ \ln\rho-\sum_{m=1}^{\infty}\dfrac{1}{m}\left(\dfrac{R}{\rho}\right)^m\cos m(\theta-\phi), & \rho>R\end{cases}$
$J_0(kr)=\begin{cases}\sum_{m=-\infty}^{\infty}J_m(kR)J_m(k\rho)\cos\big(m(\theta-\phi)\big), & R>\rho\\ \sum_{m=-\infty}^{\infty}J_m(k\rho)J_m(kR)\cos\big(m(\theta-\phi)\big), & \rho>R\end{cases}$

$Y_0(kr)=\begin{cases}\sum_{m=-\infty}^{\infty} Y_m(kR)J_m(k\rho)\cos(m(\theta-\phi)), & R>\rho \\ \sum_{m=-\infty}^{\infty} Y_m(k\rho)J_m(kR)\cos(m(\theta-\phi)), & \rho>R\end{cases}$
$H_0^{(1)}(kr)=J_0(kr)+iY_0(kr)$
$H_0^{(2)}(kr)=J_0(kr)-iY_0(kr)$
$I_0(kr)=\begin{cases}\sum_{m=-\infty}^{\infty} (-1)^m I_m(kR)I_m(k\rho)\cos(m(\theta-\phi)), & R>\rho \\ \sum_{m=-\infty}^{\infty} I_m(k\rho)I_m(kR)\cos(m(\theta-\phi)), & \rho>R\end{cases}$
$K_0(kr)=\begin{cases}\sum_{m=-\infty}^{\infty} K_m(kR)I_m(k\rho)\cos(m(\theta-\phi)), & R>\rho \\ \sum_{m=-\infty}^{\infty} K_m(k\rho)I_m(kR)\cos(m(\theta-\phi)), & \rho>R\end{cases}$
3-D
$j_0(kr)=\sqrt{\frac{\frac{1}{2}\pi}{kr}}J_{1/2}(kr)$
$y_0(kr)=\sqrt{\frac{\frac{1}{2}\pi}{kr}}Y_{1/2}(kr)$

$h_0^{(1)}(kr)=\sqrt{\frac{\frac{1}{2}\pi}{kr}}H_{1/2}^{(1)}(kr)$
$h_0^{(2)}(kr)=\sqrt{\frac{\frac{1}{2}\pi}{kr}}H_{1/2}^{(2)}(kr)$
$U(s,x)=\frac{1}{r}=\begin{cases} k\sum_{n=0}^{\infty}(2n+1)\sum_{m=0}^{n}\varepsilon_m\frac{(n-m)!}{(n+m)!}\cos\left[m(\phi-\bar{\phi})\right]P_n^m(\cos\theta)P_n^m(\cos\bar{\theta})\,j_n(k\bar{\rho})\,j_n(k\rho), \quad \bar{\rho}>\rho \\ k\sum_{n=0}^{\infty}(2n+1)\sum_{m=0}^{n}\varepsilon_m\frac{(n-m)!}{(n+m)!}\cos\left[m(\phi-\bar{\phi})\right]P_n^m(\cos\theta)P_n^m(\cos\bar{\theta})\,j_n(k\rho)\,j_n(k\bar{\rho}), \quad \rho>\bar{\rho} \end{cases}$
$U(s,x)=-\frac{e^{-ikr}}{r}=\begin{cases} ik\sum_{n=0}^{\infty}(2n+1)\sum_{m=0}^{n}\varepsilon_m\frac{(n-m)!}{(n+m)!}\cos\left[m(\phi-\bar{\phi})\right]P_n^m(\cos\theta)P_n^m(\cos\bar{\theta})\,j_n(k\rho)h_n^{(2)}(k\bar{\rho}), \quad \bar{\rho}>\rho \\ ik\sum_{n=0}^{\infty}(2n+1)\sum_{m=0}^{n}\varepsilon_m\frac{(n-m)!}{(n+m)!}\cos\left[m(\phi-\bar{\phi})\right]P_n^m(\cos\theta)P_n^m(\cos\bar{\theta})\,j_n(k\bar{\rho})h_n^{(2)}(k\rho), \quad \rho>\bar{\rho} \end{cases}$

Addition theorem by tashi