

Gaussian integral method

Analytic solution:

$$x_1 = 1;$$

$$x_2 = 0.5;$$

$$s_1 = s;$$

$$s_2 = 0;$$

$$n_1 = 0;$$

$$n_2 = -1;$$

$$\bar{n}_1 = 1;$$

$$\bar{n}_2 = 0;$$

$$y_1 = x_1 - s_1;$$

$$y_2 = x_2 - s_2;$$

$$r = \sqrt{Hs_1 - x_1 L^2 + Hs_2 - x_2 L^2};$$

$$U@sD = \text{Log}@rD;$$

$$T@sD = \frac{-Hy_1 n_1 + y_2 n_2 L}{r^2};$$

$$I@sD = \frac{y_1 \bar{n}_1 + y_2 \bar{n}_2}{r^2};$$

$$M@sD = \frac{2Hy_1 n_1 y_1 \bar{n}_1 + y_1 n_1 y_2 \bar{n}_2 + y_2 n_2 y_1 \bar{n}_1 + y_2 n_2 y_2 \bar{n}_2 L}{r^4} - \frac{n_1 \bar{n}_1 + n_2 \bar{n}_2}{r^2};$$

$$UU = \int_0^1 U@sD \hat{a}s$$

$$TT = \int_0^1 T@sD \hat{a}s$$

$$LL = \int_0^1 I@sD \hat{a}s$$

$$MM = \int_0^1 M@sD \hat{a}s$$

$$-0.334854 + 0. \ddot{a}$$

$$1.10715$$

$$0.804719$$

$$-1.6$$

高斯積分點取五項 (n=5) 時所對映權重函數之的資料表

Mapping points (0~1)	Guassian points X_i	Weighting functions W_i	Mathematica check (Exact Solution) $63x^5 - 70x^3 + 15x = 0$
9.530900E-01	9.061798E-01	2.369269E-01	-0.90618
7.692347E-01	5.384693E-01	4.786286E-01	-0.538469
5.000000E-01	0.000000E+00	5.688889E-01	0.0
2.307653E-01	-5.384693E-01	4.786286E-01	0.538469
4.691008E-02	-9.061798E-01	2.369269E-01	0.90618

Mathematica check

NSolve[63 x⁵ - 70 x³ + 15 x == 0, x] // N

88x⁰ - 0.90618<, 8x⁰ - 0.538469<,
8x⁰ 0.<, 8x⁰ 0.538469<, 8x⁰ 0.90618<<

利用 Fortran 程式所解之數值解 (Numerical Data)

U kernel: -3.348573E-01
T kernel: 1.107174
L kernel: 8.047636E-01
M kernel: -1.600755

	數值解 (Numerical Data)	解析解 (Analytic solution)
U	-3.348573E-01	-0.334854
T	1.107174	1.10715
L	8.047636E-01	0.804719
M	-1.600755	-1.6