Interior

$$u(r,f)=\frac{1}{2p}\int_0^{2p}\frac{\left(R^2-r^2\right)f(q)}{R^2+r^2-2Rr\cos(q-f)}dq$$

$$\int_0^{2p}\frac{f(q)}{1+p^2-2p\cos(q)}dq=\frac{2p}{\left(1-p^2\right)}u(r,f)$$

$$\int_0^{2p}\ln\sqrt{1+p^2-2p\cos(q)}dq=\int_0^{2p}\int_0^{p}\frac{p-\cos q}{1+p^2-2p\cos(q)}dpdq$$

$$=\int_0^{p}\int_0^{2p}\frac{p-\cos q}{1+p^2-2p\cos(q)}dq\,dp$$

$$f(q)=p,\ u(r,f)=p$$

$$\int_0^{p}\int_0^{2p}\frac{p}{1+p^2-2p\cos(q)}dq\,dp=\int_0^{p}\frac{2p\,p}{1-p^2}dp$$

$$f(q)=\cos q,\ u(r,f)=p\cos f$$

$$\int_0^{p}\int_0^{2p}\frac{\cos q}{1+p^2-2p\cos(q)}dq\,dp=\int_0^{p}\frac{2p\,p}{1-p^2}dp$$

$$\int_0^{2p}\ln\sqrt{1+p^2-2p\cos(q)}dq=0$$

Exterior

$$u(r,f)=\frac{1}{2p}\int_0^{2p}\frac{\left(r^2-R^2\right)f(q)}{R^2+r^2-2Rr\cos(q-f)}dq$$

$$\int_0^{2p}\frac{f(q)}{1+p^2-2p\cos(q)}dq=\frac{2p}{\left(p^2-1\right)}u(r,f)$$

$$\int_0^{2p}\ln\sqrt{1+p^2-2p\cos(q)}dq=\int_0^{2p}\int_0^{p}\frac{p-\cos q}{1+p^2-2p\cos(q)}dpdq$$

$$=\int_0^{p}\int_0^{2p}\frac{p-\cos q}{1+p^2-2p\cos(q)}dq\,dp$$

$$f(q)=p,\ u(r,f)=p$$

$$\int_0^{p}\int_0^{2p}\frac{p}{1+p^2-2p\cos(q)}dq\,dp=\int_0^{p}\frac{2p\,p}{p^2-1}dp$$

$$f(q)=\cos q,\ u(r,f)=\frac{1}{p}\cos f$$

$$\int_0^p \int_0^{2p} \frac{\cos q}{1+p^2-2p\cos(q)} dq\, dp = \int_0^p \frac{2p}{\left(1-p^2\right)p} dp$$

$$\int_0^{2p} \ln\sqrt{1+p^2-2p\cos(q)}\, dq = \int_0^p \frac{2p\, p}{p^2-1} dp - \int_0^p \frac{2p}{\left(p^2-1\right)p} dp$$

$$= \left[p \ln\left(p^2-1\right)\right]_0^p - 2p\left[-\ln p + \frac{1}{2}\ln\left(p^2-1\right)\right]_0^p$$

$$= 2p \ln p$$

Integral $\ln r$ using Poisson integral formula by 高聖凱