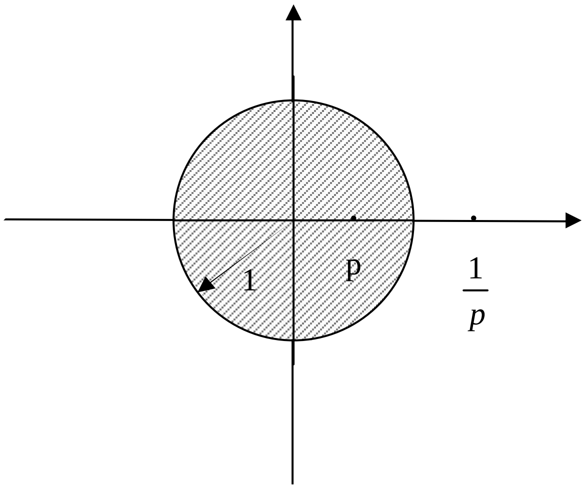
Derivation of Poisson integral formula using Cauchy integral formula



$$x = (p,0)$$
$$s = 1 \cdot e^{iq}$$

$$z = e^{iq}$$
$$dz = ie^{iq}dq$$

$$f(p) = \frac{1}{2pi}\oint\left(\frac{1}{z-p} - \frac{1}{z-\frac{1}{p}}\right)f(z)dz = \frac{1}{2pi}\oint\frac{(-\frac{1}{p}+p)}{(z^2-(p+\frac{1}{p})z+1)}f(z)dz$$

$$= \frac{1}{2pi}\oint\frac{(p-\frac{1}{p})}{(z^2-(p+\frac{1}{p})z+1)}f(z)dz = \frac{1}{2pi}\oint\frac{(p-\frac{1}{p})}{(z^2-(p+\frac{1}{p})z+1)}f(z)ie^{iq}dq$$

$$= \frac{1}{2p}\int_0^{2p}\frac{(p-\frac{1}{p})e^{iq}}{(e^{2iq}-(p+\frac{1}{p})e^{iq}+1)}f(e^{iq})dq$$

$$= \frac{1}{2p}\int_0^{2p}\frac{(p^2-1)e^{iq}e^{-iq}}{(pe^{iq}-(p^2+1)+pe^{-iq})}f(e^{iq})dq$$

$$= \frac{1}{2p}\int_0^{2p}\frac{(p^2-1)}{(2p\cos q-(p^2+1))}f(e^{iq})dq$$

$$= \frac{1}{2p}\int_0^{2p}\frac{(1-p^2)}{(1+p^2-2p\cos q)}f(e^{iq})dq$$