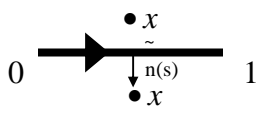


UTLM mathematica 積分

Calculate the four integrals $\int_0^1 U(s,x)t(s)ds$, $\int_0^1 T(s,x)u(s)ds$, $\int_0^1 L(s,x)t(s)ds$ and

$\int_0^1 M(s,x)u(s)ds$ by using symbolic software, where $\tilde{x} = (0.5, \epsilon)$, $\tilde{s} = (s, 0)$, $n(s) = (0, -1)$

and $n(x) = (0, -1)$. 

Please find the limiting values for $\epsilon \rightarrow 0^+$ and $\epsilon \rightarrow 0^-$.

$$\int_0^1 U(s,x)t(s)ds$$

$$\text{Limit} \left[\int_0^1 \text{Log}[\sqrt{(0.5-s)^2 + \epsilon^2}] ds, \epsilon \rightarrow 0, \text{Direction} \rightarrow 1 \right] = -1.69315$$

$$\text{Limit} \left[\int_0^1 \text{Log}[\sqrt{(0.5-s)^2 + \epsilon^2}] ds, \epsilon \rightarrow 0, \text{Direction} \rightarrow -1 \right] = -1.69315$$

$$\int_0^1 T(s,x)u(s)ds$$

$$\text{Limit} \left[\int_0^1 \frac{\epsilon}{(0.5-s)^2 + \epsilon^2} ds, \epsilon \rightarrow 0, \text{Direction} \rightarrow 1 \right] = -3.14159$$

$$\text{Limit} \left[\int_0^1 \frac{\epsilon}{(0.5-s)^2 + \epsilon^2} ds, \epsilon \rightarrow 0, \text{Direction} \rightarrow -1 \right] = 3.14159$$

$$\int_0^1 L(s,x)t(s)ds$$

$$\text{Limit} \left[\int_0^1 \frac{-\epsilon}{(0.5-s)^2 + \epsilon^2} ds, \epsilon \rightarrow 0, \text{Direction} \rightarrow 1 \right] = 3.14159$$

$$\text{Limit} \left[\int_0^1 \frac{-\epsilon}{(0.5-s)^2 + \epsilon^2} ds, \epsilon \rightarrow 0, \text{Direction} \rightarrow -1 \right] = -3.14159$$

$$\int_0^1 M(s,x)u(s)ds$$

$$\text{Limit} \left[\int_0^1 \left(\frac{2\epsilon^2}{((0.5-s)^2 + \epsilon^2)^2} - \frac{1}{(0.5-s)^2 + \epsilon^2} \right) ds, \epsilon \rightarrow 0, \text{Direction} \rightarrow 1 \right] = 4$$

$$\text{Limit} \left[\int_0^1 \left(\frac{2\epsilon^2}{((0.5-s)^2 + \epsilon^2)^2} - \frac{1}{(0.5-s)^2 + \epsilon^2} \right) ds, \epsilon \rightarrow 0, \text{Direction} \rightarrow -1 \right] = 4$$