1, $e^{2+3\pi i}$

Sol:

$$e^{2+3\pi i} = e^2 * e^{3\pi i} = -e^2$$

2, $\log(-1)$

Sol:

$$\log(-1) = \log\left(e^{(2n-1)\pi i}\right) = (2n-1)\pi i, \quad (n \in N)$$

3, $\mathrm{Log}(-1)$

Sol:

$$\log(-1) = (2n-1)\pi i$$

取$n = 0$

$$\mathrm{Log}(-1) = -\pi i$$

4, i^{-2i}

Sol:

$$i^{-2i} = e^{\left(\frac{(4n+1)\pi}{2}\right)^{-2i}} = e^{(4n+1)\pi}$$

5, Is it right $\overline{\cos(i z)} = \cos(i \bar{z})$

Sol:

$$\cos(i z) = \frac{e^{i(i z)} + e^{-i(i z)}}{2} = \frac{e^{-z} + e^{z}}{2}$$

$$\overline{\cos(i z)} = \frac{e^{-\bar{z}} + e^{\bar{z}}}{2}$$

$$\cos(i \bar{z}) = \frac{e^{i(i \bar{z})} + e^{-i(i \bar{z})}}{2} = \frac{e^{-\bar{z}} + e^{\bar{z}}}{2}$$

So

$$\overline{\cos(i z)} = \cos(i \bar{z})$$

6, Is it right $\overline{\cosh(z)} = \cos(\bar{z})$

Sol:

$$\cosh(z) = \frac{e^{-z} + e^{z}}{2}$$

$$\overline{\cosh(z)} = \frac{e^{-\bar{z}} + e^{\bar{z}}}{2}$$

$$\cos(\bar{z}) = \frac{e^{i\bar{z}} + e^{-i\bar{z}}}{2}$$

So

$$\overline{\cosh(z)} \neq \cos(\bar{z})$$

7, Prove $\tan^{-1}(z) = \frac{i}{2}\log\left(\frac{i+z}{i-z}\right)$

Sol:

let $\tan^{-1}(z) = \omega$

$$\tan(\omega) = \frac{e^{i\omega} - e^{-i\omega}}{i\left(e^{i\omega} + e^{-i\omega}\right)} = z$$

$$e^{i\omega} - e^{-i\omega} = z\, i\left(e^{i\omega} + e^{-i\omega}\right)$$

$$(1 - z\, i)\, e^{2 i\omega} = (1 + z\, i)$$

$$e^{i\omega} = \left(\frac{1 + z\, i}{1 - z\, i}\right)^{\frac{1}{2}}$$

$$i\,\omega = \frac{1}{2}\log\left(\frac{1 + z\, i}{1 - z\, i}\right)$$

$$\omega = \frac{i}{2}\log\left(\frac{i + z}{i - z}\right)$$

So

$$\tan^{-1}(z) = \omega = \frac{i}{2}\log\left(\frac{i + z}{i - z}\right)$$