

1.

$$Z_1 = 0 + 2i$$

$$Z_2 = 1 - i$$

$$Z_3 = \frac{1}{2} + \frac{1}{2}i$$

$$Z_4 = \frac{3}{4} - \frac{i}{4}$$

$$Z_5 = \frac{5}{8} + \frac{i}{8}$$

$$Z_6 = \frac{11}{16} - \frac{i}{16}$$

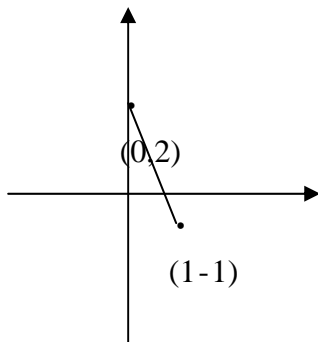
$$Z_7 = \frac{21}{32} + \frac{i}{32}$$

$$Z_8 = \frac{43}{64} - \frac{i}{64}$$

由虛部可知數列為 $2, -1, \frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \frac{1}{32}, -\frac{1}{64}, \dots$

故此數列第 n 項為 $2(-\frac{1}{2})^n$

$$\rightarrow \lim_{n \rightarrow \infty} 2(-\frac{1}{2})^n = 0 \quad \text{虛部收斂至 } 0$$



由幾何關係知直線方程式為 $y = -3x + 2$

$$\rightarrow y = 0, x = \frac{2}{3} \quad \text{得實部收斂到 } \frac{2}{3}$$

$$\text{故 } \lim_{n \rightarrow \infty} Z_n = \lim_{n \rightarrow \infty} X_n + i \lim_{n \rightarrow \infty} Y_n = \frac{2}{3}$$

2.

$$Z_1 = 0 + 2i$$

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$$Z_8 = \frac{43}{64} - \frac{i}{64}$$

由實部可知

$$\begin{array}{cccccccc}
 \sum_{N=0}^0 \left(-\frac{1}{2}\right)^N & \sum_{N=0}^1 \left(-\frac{1}{2}\right)^N & \sum_{N=0}^2 \left(-\frac{1}{2}\right)^N & \sum_{N=0}^3 \left(-\frac{1}{2}\right)^N & \sum_{N=0}^4 \left(-\frac{1}{2}\right)^N & \sum_{N=0}^5 \left(-\frac{1}{2}\right)^N & \sum_{N=0}^6 \left(-\frac{1}{2}\right)^N & \dots\dots\dots \\
 \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \\
 0, 1\dots & , & \frac{1}{2}\dots & , & \frac{3}{4}\dots & , & \frac{5}{8}\dots & , & \frac{11}{16} & , & \dots\frac{21}{32}\dots & , & \frac{43}{64} & \dots\dots\dots \\
 \vee & \vee & & \vee & \vee & & \vee & \vee & & \vee & & \vee & & \dots\dots\dots \\
 1 & -\frac{1}{2} & & \frac{1}{4} & -\frac{1}{8} & & \frac{1}{16} & -\frac{1}{32} & & \frac{1}{64} & & & & \dots\dots\dots
 \end{array}$$

故此數列第 n 項為 $\sum_{N=0}^{\infty} \left(-\frac{1}{2}\right)^N, N = n - 1$

$$\lim_{n \rightarrow \infty} \sum_{N=0}^{n-1} \left(-\frac{1}{2}\right)^N = \sum_{n=0}^{\infty} \left(-\frac{1}{2}\right)^n = \frac{1}{1 - \left(-\frac{1}{2}\right)} = \frac{2}{3}$$

實部收斂至 $\frac{2}{3}$

$$\text{故 } \lim_{n \rightarrow \infty} Z_n = \lim_{n \rightarrow \infty} X_n + i \lim_{n \rightarrow \infty} Y_n = \frac{2}{3}$$