

Expand $f(z) = \frac{1}{z^2(1-z)}$ into Laurent series

(1) $0 < |z| < 1$

$$f(z) = \frac{1}{z^2} \cdot \frac{1}{1-z} = \frac{1}{z^2} \sum_{n=0}^{\infty} z^n = \sum_{n=0}^{\infty} z^{n-2}$$

(2) $1 < |z| < \infty$

$$f(z) = \frac{1}{z^2} \cdot \frac{1}{1-z} = \frac{-1}{z^3} \cdot \frac{1}{1-\frac{1}{z}} = \frac{-1}{z^3} \sum_{n=0}^{\infty} \left(\frac{1}{z}\right)^n = - \sum_{n=0}^{\infty} \frac{1}{z^{n+3}}$$

