Check HW9.

$$F_I(\omega)=\frac{1}{\sqrt{2\pi}}\frac{\omega^3}{4+\omega^4}$$

$$F_R(\omega)=\frac{1}{\sqrt{2\pi}}\frac{2+\omega^2}{4+\omega^4}$$

$$\rightarrow \frac{1}{\sqrt{2\pi}}\frac{\omega^3}{4+\omega^4}*\frac{\sqrt{2}}{\sqrt{\pi}}\frac{-i}{\omega}=\frac{-i}{\pi}\int_{-\infty}^{\infty}\frac{u^3}{4+u^4}\frac{1}{t-u}du=\frac{-i}{\pi}\int_{-\infty}^{\infty}f(u)du$$

$$Resf(-1+i)=\frac{1}{(4-4i)+4\omega}\qquad Resf(1+i)=\frac{1}{(-4-4i)+4\omega}\qquad Resf(\omega)=\frac{\omega^3}{4+\omega^4}$$

$$\int_{-\infty}^{\infty}\frac{u^3}{4+u^4}\frac{1}{t-u}du=2\pi i[Res(-1+i)+Res(1+i)]-\pi i[Res(\omega)]=-\pi\frac{2+\omega^2}{4+\omega^4}$$

$$\rightarrow \frac{1}{\sqrt{2\pi}}\cdot\frac{1}{\pi}\cdot-\pi\frac{2+\omega^2}{4+\omega^4}=-\frac{1}{\sqrt{2\pi}}\frac{2+\omega^2}{4+\omega^4}$$

$$\boxed{\therefore \frac{1}{\sqrt{2\pi}}[F_I(\omega)*\sqrt{\frac{2}{\pi}}(\frac{1}{\omega})]=-F_R(\omega)}$$

Hilbert transform

$$F_R(\omega)=\frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{F_I(u)}{\omega-u}du=\frac{1}{\sqrt{2\pi}}\frac{2+\omega^2}{4+\omega^4}$$

Check HW10.

$$F_I(\omega)=\frac{\omega}{\sqrt{2\pi}(\omega^2-1)}$$

$$F_R(\omega)=\frac{\sqrt{\pi}}{2\sqrt{2}}[\delta(\omega-1)+\delta(\omega+1)]$$

$$\frac{1}{\sqrt{2\pi}}\frac{\omega}{\omega^2-1}*\frac{\sqrt{2}}{\sqrt{\pi}}(\frac{-i}{\omega})=\frac{-i}{\pi}\int_{-\infty}^{\infty}\frac{u}{u^2-1}\frac{1}{\omega-u}du=\frac{-i}{2\pi}\int_{-\infty}^{\infty}(\frac{1}{u-1}+\frac{1}{u+1})\frac{1}{\omega-u}du$$

∵Hilbert transform

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\delta(u+1)}{\omega-u}du=\frac{1}{\pi}\frac{1}{\omega+1}$$

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\delta(u-1)}{\omega-u}du=\frac{1}{\pi}\frac{1}{\omega-1}$$

$$H\{H\{f(\omega)\}\}=-f(\omega)$$

$$\therefore\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{\pi}\frac{1}{\omega+1}\frac{1}{\omega-u}d\omega=-\delta(\omega+1)$$

$$\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{\pi}\frac{1}{\omega-1}\frac{1}{\omega-u}d\omega=-\delta(\omega-1)$$

$$\therefore\int_{-\infty}^{\infty}(\frac{1}{u-1}+\frac{1}{u+1})\frac{1}{\omega-u}du=-\pi^2(\delta(\omega+1)+\delta(\omega-1))$$

$$\rightarrow\frac{1}{\sqrt{2\pi}}\cdot\frac{1}{2\pi}\cdot-\pi^2(\delta(\omega+1)+\delta(\omega-1))=\frac{-\pi}{2\sqrt{2}}(\delta(\omega+1)+\delta(\omega-1))$$

Hilbert transform

$$F_R(\omega)=\frac{-1}{\pi}\int_{-\infty}^{\infty}\frac{F_I(u)}{\omega-u}du=\frac{\sqrt{\pi}}{2\sqrt{2}}[\delta(\omega-1)+\delta(\omega+1)]$$