

$$u(\rho, \phi) = u(p) = \frac{1}{2\pi i} \oint \left( \frac{1}{z-p} - \frac{1}{z-\frac{1}{p}} \right) f(z) dz$$

$$= \frac{1}{2\pi i} \oint \left( \frac{p-\frac{1}{p}}{(z-\frac{1}{p})(z-p)} \right) f(z) dz$$

$$= \frac{1}{2\pi i} \oint \left( \frac{p-\frac{1}{p}}{z^2 - (\frac{1}{p}+p)z + 1} \frac{p}{z} \right) f(z) dz$$

$$= \frac{1}{2\pi i} \oint \left( \frac{p^2-1}{(z+\frac{1}{z})p - (p^2+1)} \right) f(z) \frac{dz}{z}$$

$$= \frac{1}{2\pi i} \int_{c_1+c_2+c_3+c_4} \left( \frac{p^2-1}{(z+\frac{1}{z})p - (p^2+1)} \right) f(z) \frac{dz}{z}$$

$$= \frac{1}{2\pi i} \int_{c_2} \left( \frac{p^2-1}{(z+\frac{1}{z})p - (p^2+1)} \right) f(z) \frac{dz}{z}$$

$$= \frac{1}{2\pi i} \oint \left( \frac{p^2-1}{(z+\frac{1}{z})p - (p^2+1)} \right) f(z) \frac{dz}{z}$$

$$\because z = e^{i\theta}, dz = ie^{i\theta} d\theta$$

$$u(\rho, \phi) = \frac{1}{2\pi i} \oint \left( \frac{p^2-1}{(z+\frac{1}{z})p - (p^2+1)} \right) f(z) \frac{dz}{z}$$

$$= \frac{1}{2\pi} \int_{2\pi}^0 \left( \frac{p^2-1}{-(p^2+1) + 2p \cos \theta} \right) f(\theta) d\theta$$

$$= \frac{1}{2\pi} \int_{2\pi}^0 \left( \frac{1-p^2}{(p^2+1) - 2p \cos \theta} \right) f(\theta) d\theta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} \left( \frac{p^2-1}{(p^2+1) - 2p \cos \theta} \right) f(\theta) d\theta$$

