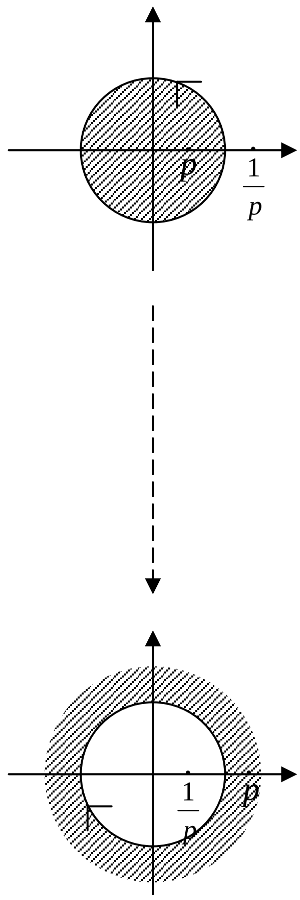
內外域 Poisson 積分公式

Poisson integral formula(interior case)

$$f(p)=\frac{1}{2\pi}\oint\left(\frac{1}{\xi-p}-\frac{1}{\xi-\frac{1}{p}}\right)f(\xi)d\xi$$

$$=\frac{1}{2\pi}\int_{0}^{2\pi}\frac{(1-p^2)}{1+p^2-2p\cos\theta}f(\theta)d\theta$$



By mapping

令 $\xi=\frac{1}{z}=e^{-i\theta}$

$d\xi=-ie^{-i\theta}d\theta$

Poisson integral formula(exterior case)

$$f\left(\frac{1}{p}\right)=\frac{1}{2\pi}\int_{2\pi}^{0}\frac{1-\left(\frac{1}{p}\right)^2}{1+\left(\frac{1}{p}\right)^2-2\left(\frac{1}{p}\right)\cos\theta}f(\theta)(-d\theta)$$

$$=\frac{1}{2\pi}\int_{2\pi}^{0}\frac{p^2-1}{p^2+1-2p\cos\theta}f(\theta)(-d\theta)$$

$$=\frac{1}{2\pi}\int_{2\pi}^{0}\frac{1-p^2}{p^2+1-2p\cos\theta}f(\theta)d\theta$$

$$=\frac{1}{2\pi}\int_{0}^{2\pi}\frac{p^2-1}{p^2+1-2p\cos\theta}f(\theta)d\theta$$