邊界元素法作業

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I. Dual integral equations for complex variables on a domain point:

$$W(z) = u(x,y) + iv(x,y) = \frac{1}{2\pi i}\int_B \frac{W(t)}{t-z}dt \tag{1}$$

$$W'(z) = \frac{1}{2\pi i}\int_B \frac{W(t)}{(t-z)^2}dt \tag{2}$$

Dual integral equations for complex variables on a boundary point:

$$W(z) = u(x,y) + iv(x,y) = \frac{1}{2}W(z) + \frac{1}{2\pi i}CPV\int_B \frac{W(t)}{t-z}dt \tag{3}$$

$$W'(z) = \frac{1}{2\pi i}HPV\int_B \frac{W(t)}{(t-z)^2}dt \tag{4}$$

(a). If $W(z)$ is analytic function, then u, v are Cauchy Riemann pairs.

(b). Based on Eqs.(1) and (2), derive the dual integral equations for real variables on a domain point

$$2\pi u(x) = \int_B \{T(s,x)u(s) - U(s,x)t(s)\}dB(s)$$

$$2\pi t(x) = \int_B \{M(s,x)u(s) - L(s,x)t(s)\}dB(s)$$

Based on Eqs.(3) and (4), derive the dual integral equations for real variables on a boundary point

$$\pi u(x) = CPV\int_B T(s,x)u(s)dB(s) - RPV\int_B U(s,x)t(s)dB(s)$$

$$\pi t(x) = HPV\int_B M(s,x)u(s)dB(s) - CPV\int_B L(s,x)t(s)dB(s)$$

(c). Discuss the Cauchy principal value as z approaches B.

(d). Discuss the Hadamard principal value as z approaches B.

(e). Proof of the following identities.

$$2\pi = \int_B \{T(s,x)\}dB(s)$$

$$0 = \int_B \{M(s,x)\}dB(s)$$