國立臺灣海洋大學河海工程學系 1997 工程數學（三）第十次作業

1. Determine the following integral

$$\int_C \frac{1}{z} dz$$

   where C is the line element from (r_1, θ_1) to (r_2, θ_2) in terms of polar coordinate.

(a). Proof by complex method for

$$\int_C \frac{1}{z} dz = ln(r_2) - ln(r_1) + i(\theta_2 - \theta_1)$$

(b). Proof by real method using the line element (x_1, y_1) to (x_2, y_2)

   $z = x + yi$

   $dz = dx + dyi$

   $x_1 = r_1 cos(\theta_1)$

   $y_1 = r_1 sin(\theta_1)$

   $x_2 = r_2 cos(\theta_2)$

   $y_2 = r_2 sin(\theta_2)$

(c). Determine the solution for the special case for the line element $(r_1, 0)$ to $(r_2, 0)$ in terms of polar coordinate.

(d). Determine the solution for the special case for the line element (r_1, π) to (r_2, π) in terms of polar coordinate.

(e). Determine the solution for the special case for the line element (r_1, π) to $(r_2, 0)$ in terms of polar coordinate.