

$$(x, y) \rightarrow (z, \bar{z})$$

海大河海系

陳正宗

1 Two independent variables

$$(x, y) \rightarrow (z, \bar{z})$$

2 Relations of two independent variables

$$x = \frac{z + \bar{z}}{2}, \quad z = x + iy$$

$$y = \frac{z - \bar{z}}{2i}, \quad z = x - iy$$

3 Two dependent variables

$$u(x, y), v(x, y) \rightarrow U(z, \bar{z}), V(z, \bar{z})$$

4 Single derivative

$$\frac{\partial}{\partial z} = \frac{1}{2} \left\{ \frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right\}$$

$$\frac{\partial}{\partial \bar{z}} = \frac{1}{2} \left\{ \frac{\partial}{\partial x} + i \frac{\partial}{\partial y} \right\}$$

5 Single derivative

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial z} \frac{\partial z}{\partial x} + \frac{\partial}{\partial \bar{z}} \frac{\partial \bar{z}}{\partial x} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \bar{z}}$$

$$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} \frac{\partial z}{\partial y} + \frac{\partial}{\partial \bar{z}} \frac{\partial \bar{z}}{\partial y} = i \left\{ \frac{\partial}{\partial z} - \frac{\partial}{\partial \bar{z}} \right\}$$

6 Double derivative

$$\frac{\partial^2}{\partial x^2} = \left\{ \frac{\partial^2}{\partial z^2} + 2 \frac{\partial^2}{\partial z \partial \bar{z}} + \frac{\partial^2}{\partial \bar{z}^2} \right\}$$

$$\frac{\partial^2}{\partial y^2} = - \left\{ \frac{\partial^2}{\partial z^2} - 2 \frac{\partial^2}{\partial z \partial \bar{z}} + \frac{\partial^2}{\partial \bar{z}^2} \right\}$$

7 Laplace operator

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z \partial \bar{z}}$$

8 Cauchy-Riemann equation

$$f(z, \bar{z}) = u(x, y) + v(x, y)i$$

If

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Implies

$$\frac{\partial f}{\partial x} = -i \frac{\partial f}{\partial y}, \quad \rightarrow \quad \frac{\partial f}{\partial \bar{z}} = 0 \quad \rightarrow \quad f(z) \text{ only}$$