(1)

(a)

$x^3 - 6x^2 + 11x - 6 = 0$

Let $x = y + 2$

$y^3 - y = 0$

Let $y = s \times z$

$z^3 - \frac{1}{s^2} z =$

$0 \to (\text{Sin}[\theta])^3 - \frac{3}{4}\text{Sin}[\theta] = -\frac{1}{4}\text{Sin}[3\theta]$

When $s = \frac{2}{\sqrt{3}}$

Let $z = \text{Sin}[\theta]$

$\text{Sin}[3\theta] = 0$

$3\theta = 2n\pi$

$\theta = \frac{2}{3}n\pi, \ \theta = 0, \ \frac{2\pi}{3}, \ \frac{4\pi}{3}$

$\text{Sin}[\theta] = 0, \ \frac{\sqrt{3}}{2}, \ -\frac{\sqrt{3}}{2}$

$y = s \times z = 0, \ 1, \ -1$

$x = 2, \ 3, \ 1$

When $s = -\frac{2}{\sqrt{3}}$

Let $z = \text{Sin}[\theta]$

$\text{Sin}[3\theta] = 0$

$$3\theta = 2n\pi$$

$$\theta = \frac{2}{3} n\pi, \quad \theta = 0, \quad \frac{2\pi}{3}, \quad \frac{4\pi}{3}$$

$$\text{Sin}[\theta] = 0, \quad \frac{\sqrt{3}}{2}, \quad -\frac{\sqrt{3}}{2}$$

$$y = s \times z = 0, \ 1, \ -1$$

$$x = 2, \ 1, \ 3$$

(b)

$$x^3 + 2x^2 + 2x + 1 = 0$$

Let $x = y - \frac{2}{3}$

$$y^3 + \frac{2y}{3} + \frac{7}{27} = 0 \to (\text{Sin}[\theta])^3 - \frac{3}{4}\text{Sin}[\theta] = -\frac{1}{4}\text{Sin}[3\theta]$$

Let $y = s \times z$

$$z^3 + \frac{2z}{3s^2} + \frac{7}{27s^3} = 0$$

Let $\frac{2}{3s^2} = -\frac{3}{4}$

When $s = \frac{2\sqrt{2}\, i}{3}$

$$z^3 - \frac{3}{4}z + \frac{7}{27s^3} = 0 \to (\text{Sin}[\theta])^3 - \frac{3}{4}\text{Sin}[\theta] = -\frac{1}{4}\text{Sin}[3\theta]$$

Let $z = \mathrm{Sin}[\theta]$

$$\mathrm{Sin}[3\theta] = \frac{7\sqrt{2}}{8}\, i$$

Let $3\theta = 3\alpha + 3\beta\, i$

$$\mathrm{Sin}[3\theta] = \mathrm{Sin}[3\alpha]\,\mathrm{Cosh}[3\beta] + i\,\mathrm{Cos}[3\alpha]\,\mathrm{Sinh}[3\beta] = \frac{7\sqrt{2}}{8}\, i$$

$\mathrm{Sin}[3\alpha]\,\mathrm{Cosh}[3\beta] = 0,$

$$\mathrm{Cos}[3\alpha]\,\mathrm{Sinh}[3\beta] = \frac{7\sqrt{2}}{8}$$

Assume $\mathrm{Sin}[3\alpha] = 0$

$$\alpha = \frac{2n\pi}{3} = 0,\quad \frac{2\pi}{3},\quad \frac{4\pi}{3}$$

$$\mathrm{Sinh}[3\beta] = \frac{7\sqrt{2}}{8}$$

Let $e^{3\beta} = a$

$$a - \frac{1}{a} = \frac{7\sqrt{2}}{4}$$

$a = 2\sqrt{2}$

$$\beta = \mathrm{Log}\left[a^{\frac{1}{3}}\right] = \mathrm{Log}\left[\sqrt{2}\right]$$

$$\theta = i \times \mathrm{Log}\left[\sqrt{2}\right]$$

$$= \frac{2\pi}{3} + i \times \mathrm{Log}\left[\sqrt{2}\right]$$

$$= \frac{4\pi}{3} + i \times \mathrm{Log}\left[\sqrt{2}\right]$$

$$z = \mathrm{Sin}[\theta] = \mathrm{Sin}\left[i \times \mathrm{Log}\left[\sqrt{2}\right]\right] = \frac{i}{2\sqrt{2}}$$

$$=$$

$$\mathrm{Sin}\left[\frac{2\pi}{3} + i \times \mathrm{Log}\left[\sqrt{2}\right]\right] = \frac{1}{4}\sqrt{13 - 3\, i\sqrt{3}}$$

$$= \mathrm{Sin}\left[\frac{4\pi}{3} + i \times \mathrm{Log}\left[\sqrt{2}\right]\right] =$$

$$-\frac{3\sqrt{\frac{3}{2}}}{4} - \frac{i}{4\sqrt{2}}$$

$$y = s \times z = \left(\frac{i}{2\sqrt{2}}\right) * \frac{2\sqrt{2}\, i}{3} = -\frac{1}{3}$$

$$=$$

$$\left(\frac{1}{4}\sqrt{13 - 3\, i\sqrt{3}}\right) * \frac{2\sqrt{2}\, i}{3} = \frac{1}{6}\left(1 + 3\, i\sqrt{3}\right)$$

$$= \left(-\frac{3\sqrt{\frac{3}{2}}}{4} - \frac{i}{4\sqrt{2}}\right) * \frac{2\sqrt{2}\, i}{3} =$$

$$\frac{1}{6}\left(1 - 3\, i\sqrt{3}\right)$$

$$x = y - \frac{2}{3} = -\frac{1}{3} - \frac{2}{3} = -1$$

$$= \frac{1}{6}\left(1 + 3\, i\sqrt{3}\right) - \frac{2}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2}\, i$$

$$= \frac{1}{6}\left(1 - 3\,i\sqrt{3}\right) - \frac{2}{3} = -\frac{1}{2} - \frac{\sqrt{3}}{2}\,i$$

When $s = -\frac{2\sqrt{2}\,i}{3}$

$$z^3 - \frac{3}{4}z + \frac{7}{27 s^3} =$$

$$0 \to (\mathrm{Sin}[\theta])^3 - \frac{3}{4}\mathrm{Sin}[\theta] = -\frac{1}{4}\mathrm{Sin}[3\theta]$$

Let $z = \mathrm{Sin}[\theta]$

$$\mathrm{Sin}[3\theta] = -\frac{7\sqrt{2}}{8}\,i$$

Let $3\theta = 3\alpha + 3\beta\,i$

$$\mathrm{Sin}[3\theta] = \mathrm{Sin}[3\alpha]\,\mathrm{Cosh}[3\beta] + i\,\mathrm{Cos}[3\alpha]\,\mathrm{Sinh}[3\beta] = -\frac{7\sqrt{2}}{8}\,i$$

$\mathrm{Sin}[3\alpha]\,\mathrm{Cosh}[3\beta] = 0,$

$$\mathrm{Cos}[3\alpha]\,\mathrm{Sinh}[3\beta] = -\frac{7\sqrt{2}}{8}$$

Assume $\mathrm{Sin}[3\alpha] = 0$

$$\alpha = \frac{2n\pi}{3} = 0,\quad \frac{2\pi}{3},\quad \frac{4\pi}{3}$$

$$\mathrm{Sinh}[3\beta] = -\frac{7\sqrt{2}}{8}$$

Let $e^{3\beta} = a$

$$a - \frac{1}{a} = -\frac{7\sqrt{2}}{4}$$

$$a = \frac{\sqrt{2}}{4}$$

$$\beta = \mathrm{Log}\left[a^{\frac{1}{3}}\right] = \mathrm{Log}\left[\frac{1}{\sqrt{2}}\right]$$

$$\theta = i \times \mathrm{Log}\left[\frac{1}{\sqrt{2}}\right]$$

$$= \frac{2\pi}{3} + i \times \mathrm{Log}\left[\frac{1}{\sqrt{2}}\right]$$

$$= \frac{4\pi}{3} + i \times \mathrm{Log}\left[\frac{1}{\sqrt{2}}\right]$$

$$z = \mathrm{Sin}[\theta] = \mathrm{Sin}\left[i \times \mathrm{Log}\left[\frac{1}{\sqrt{2}}\right]\right] = \frac{-i}{2\sqrt{2}}$$

$$=$$

$$\mathrm{Sin}\left[\frac{2\pi}{3} + i \times \mathrm{Log}\left[\frac{1}{\sqrt{2}}\right]\right] = \frac{-1}{4}\sqrt{13 - 3\,i\sqrt{3}}$$

$$= \mathrm{Sin}\left[\frac{4\pi}{3} + i \times \mathrm{Log}\left[\frac{1}{\sqrt{2}}\right]\right] =$$

$$\frac{3\sqrt{\frac{3}{2}}}{4} + \frac{i}{4\sqrt{2}}$$

$$y = s \times z = \left(\frac{-i}{2\sqrt{2}}\right) * \frac{-2\sqrt{2}\, i}{3} = -\frac{1}{3}$$

$$= \left(\frac{-1}{4}\sqrt{13 - 3 i \sqrt{3}}\right) * \frac{-2\sqrt{2}\, i}{3} =$$

$$\frac{1}{6}\left(1 + 3 i \sqrt{3}\right)$$

$$= \left(\frac{3\sqrt{\frac{3}{2}}}{4} + \frac{i}{4\sqrt{2}}\right) * \frac{-2\sqrt{2}\, i}{3} =$$

$$\frac{1}{6}\left(1 - 3 i \sqrt{3}\right)$$

$$x = y - \frac{2}{3} = -\frac{1}{3} - \frac{2}{3} = -1$$

$$= \frac{1}{6}\left(1 + 3 i \sqrt{3}\right) - \frac{2}{3} = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$$

$$=$$

$$\frac{1}{6}\left(1 - 3 i \sqrt{3}\right) - \frac{2}{3} = -\frac{1}{2} + -\frac{\sqrt{3}}{2} i$$

(2)

$z = x + y\,i$

(a)

$\text{Cosh}[x + y\,i] =$

$\text{Cosh}[x]\,\text{Cos}[y] + i\,\text{Sinh}[x]\,\text{Sin}[y]$

$\text{Conjugate}[\text{Cosh}[x + y\,i]] =$

$\text{Cosh}[x]\,\text{Cos}[y] - i\,\text{Sinh}[x]\,\text{Sin}[y]$

$\text{Cosh}[\text{Conjugate}[x + y\,i]] = \text{Cosh}[x - y\,i] =$

$\text{Cosh}[x]\,\text{Cos}[y] - i\,\text{Sinh}[x]\,\text{Sin}[y]$

$\text{Conjugate}[\text{Cosh}[x + y\,i]] =$

$\text{Cosh}[\text{Conjugate}[x + y\,i]]$

It's right

(b)

$\text{Sinh}[x + y\,i] =$

$\text{Sinh}[x]\,\text{Cos}[y] + i\,\text{Cosh}[x]\,\text{Sin}[y]$

$\text{Conjugate}[\text{Sinh}[x + y\,i]] =$

$\text{Sinh}[x]\,\text{Cos}[y] - i\,\text{Cosh}[x]\,\text{Sin}[y]$

$\text{Sinh}[\text{Conjugate}[x + y\,i]] = \text{Sinh}[x - y\,i] =$

$\text{Sinh}[x]\,\text{Cos}[y] - i\,\text{Cosh}[x]\,\text{Sin}[y]$

$\text{Conjugate}[\text{Sinh}[x + y\,i]] =$

$\text{Sinh}[\text{Conjugate}[x + y\,i]]$

It's right

(3)

$z = e^{i*\theta}$

$$\text{Cos}[\theta] = \frac{1}{2}\left(z + \frac{1}{z}\right)$$

$$\mathrm{Sin}[\theta] = \frac{1}{2 * i}\left(z - \frac{1}{z}\right)$$

$$d\theta = \frac{1}{i * z}\, dz$$

(a)

$$\int_0^{2\pi} \frac{1}{1 + p^2 - 2 * p * \mathrm{Cos}[\theta]}\, d\theta =$$

$$\int_0^{2\pi} \frac{1}{1 + p^2 - 2 * p * \frac{1}{2}\left(z + \frac{1}{z}\right)}\, \frac{1}{i * z}\, dz =$$

$$\int_0^{2\pi} \frac{-\frac{1}{i * p}}{(z - p) * \left(z - \frac{1}{p}\right)}\, dz$$

When $z = p$, $|p| < 1$

$$\int_0^{2\pi} \frac{-\frac{1}{i * p}}{(z - p) * \left(z - \frac{1}{p}\right)}\, dz =$$

$$2\pi i \left(\frac{-\frac{1}{i * p}}{\left(p - \frac{1}{p}\right)}\right) = \frac{2\pi}{1 - p^2}$$

When $z = \frac{1}{p}$, $|p| > 1$

$$\int_0^{2\pi} \frac{-\frac{1}{i * p}}{(z - p) * \left(z - \frac{1}{p}\right)}\, dz =$$

$$2\pi i\left(\frac{-\frac{1}{i*p}}{\left(\frac{1}{p}-p\right)}\right)=\frac{2\pi}{p^2-1}$$

(b)

$$\int_0^{2\pi}\frac{-\frac{1}{i*p}*\frac{1}{2}*(z^2+\frac{1}{z^2})}{(z-p)*\left(z-\frac{1}{p}\right)}dz=$$

$$\int_0^{2\pi}\frac{-1*(z^4+1)}{2\,i\,p*z^2*(z-p)*\left(z-\frac{1}{p}\right)}dz$$

When $|p|<1,\ z=0,\ z=p$

$$\int_0^{2\pi}\frac{-1*(z^4+1)}{2\,i\,p*z^2*(z-p)*\left(z-\frac{1}{p}\right)}dz=$$

$$2\pi i\left(\text{Limit}\left[\frac{d}{dz}\left(\frac{-1*(z^4+1)}{2\,i\,p*(z-p)*\left(z-\frac{1}{p}\right)}\right),\ z\to 0\right]+\frac{-1*(p^4+1)}{2\,i\,p*p^2*\left(p-\frac{1}{p}\right)}\right)=\frac{2p^2\pi}{1-p^2}$$

When $|p|>1,\ z=0,\ z=\frac{1}{p}$

$$\int_0^{2\pi}\frac{-1*(z^4+1)}{2\,i\,p*z^2*(z-p)*\left(z-\frac{1}{p}\right)}dz=$$

$$2\pi i\left(\text{Limit}\left[\frac{d}{dz}\left(\frac{-1*(z^4+1)}{2\,i\,p*(z-p)*\left(z-\frac{1}{p}\right)}\right),\; z\to 0\right]+\frac{-1*\left(\left(\frac{1}{p}\right)^4+1\right)}{2\,i\,p*\left(\frac{1}{p}\right)^2*\left(\frac{1}{p}-p\right)}\right)=\frac{2\pi}{-p^2+p^4}$$

(c)

$$\int_0^{2\pi}\frac{-\frac{1}{i*p}*\left(1-p*\frac{z+\frac{1}{z}}{2}\right)}{(z-p)*\left(z-\frac{1}{p}\right)}\,dz=$$

$$\int_0^{2\pi}\frac{-1*\left(z-\frac{p}{2}*(z^2+1)\right)}{i\,p*z*(z-p)*\left(z-\frac{1}{p}\right)}\,dz$$

When $|p|<1$, $z=0$, $z=p$

$$\int_0^{2\pi}\frac{-1*\left(z-\frac{p}{2}*(z^2+1)\right)}{i\,p*z*(z-p)*\left(z-\frac{1}{p}\right)}\,dz=$$

$$2\pi i\left(\frac{-1*\left(0-\frac{p}{2}*(0^2+1)\right)}{i\,p*(0-p)*\left(0-\frac{1}{p}\right)}+\frac{-1*\left(p-\frac{p}{2}*(p^2+1)\right)}{i\,p*p*\left(p-\frac{1}{p}\right)}\right)=2\pi$$

When $|p|>1$, $z=0$, $z=\frac{1}{p}$

$$\int_0^{2\pi} \frac{-1*(z-\frac{p}{2}*(z^2+1))}{i\,p*z*(z-p)*\left(z-\frac{1}{p}\right)}\,dz =$$

$$2\pi i\left(\frac{-1*(0-\frac{p}{2}*(0^2+1))}{i\,p*(0-p)*\left(0-\frac{1}{p}\right)} + \frac{-1*\left(\frac{1}{p}-\frac{p}{2}*\left(\left(\frac{1}{p}\right)^2+1\right)\right)}{i\,p*\frac{1}{p}*\left(\frac{1}{p}-p\right)}\right) = 0$$

(d)

$$\int_0^{2\pi} \frac{-\frac{1}{i*p}*\frac{1}{2*i}*(z-\frac{1}{z})}{(z-p)*\left(z-\frac{1}{p}\right)}\,dz =$$

$$\int_0^{2\pi} \frac{(z^2-1)}{2*p*z*(z-p)*\left(z-\frac{1}{p}\right)}\,dz$$

When $|p| < 1$, $z = 0$, $z = p$

$$\int_0^{2\pi} \frac{(z^2-1)}{2*p*z*(z-p)*\left(z-\frac{1}{p}\right)}\,dz =$$

$$2\pi i\left(\frac{(0^2-1)}{2*p*(0-p)*\left(0-\frac{1}{p}\right)} + \frac{(p^2-1)}{2*p*p*\left(p-\frac{1}{p}\right)}\right) = 0$$

When $|p| > 1,\ z = 0,\ z = \frac{1}{p}$

$$\int_0^{2\pi} \frac{(z^2 - 1)}{2 * p * z * (z - p) * \left(z - \frac{1}{p}\right)}\, dz =$$

$$2\pi i \left(\frac{(0^2 - 1)}{2 * p * (0 - p) * \left(0 - \frac{1}{p}\right)} + \frac{\left(\left(\frac{1}{p}\right)^2 - 1\right)}{2 * p * \frac{1}{p} * \left(\frac{1}{p} - p\right)} \right) = 0$$

(4)

Assume $z[t] = f[t]\, e^{i*t}$,

$z'[t] = f'[t]\, e^{i*t} + i\, f[t]\, e^{i*t}$,

$z''[t] = f''[t]\, e^{i*t} + 2\, i\, f'[t]\, e^{i*t} - f[t]\, e^{i*t}$

$z''[t] + z[t] = f''[t]\, e^{i*t} + 2\, i\, f'[t]\, e^{i*t} = e^{i*t} \to f''[t] + 2\, i\, f'[t] = 1$

$\to (f'[t]\, e^{2*i*t})' = e^{2*i*t}$

$$f'[t] = \frac{1}{2\, i} + c_1\, e^{-2*i*t}$$

$$f[t] = \frac{t}{2\, i} + \frac{c_1 * e^{-2*i*t}}{2\, i} + c_2$$

Set $c_1 = c_2 = 0$

$$z[t] = \frac{t}{2\, i} * e^{i*t}$$

(5)

$$e^{\frac{\pi * i}{2}} = \mathrm{Cos}\left[\frac{\pi}{2}\right] + i * \mathrm{Sin}\left[\frac{\pi}{2}\right] = i$$

$$\log[i] = \log\left[e^{\frac{\pi * i}{2}}\right] = \frac{\pi * i}{2}$$

(6)

(a)

$$u^2 + v^2 = \frac{1}{4}$$

$$w = \frac{z + \frac{1}{4}}{z + 4} = \frac{x + y\, i + \frac{1}{4}}{x + y\, i + 4} =$$

$$\frac{x^2 + \frac{17}{4} * x + y^2 + 1}{(x + 4)^2 + y^2} + \frac{\frac{15}{4} * y}{(x + 4)^2 + y^2}\, i = u + v\, i$$

$$u = \frac{x^2 + \frac{17}{4} * x + y^2 + 1}{(x + 4)^2 + y^2}$$

$$v = \frac{\frac{15}{4} * y}{(x + 4)^2 + y^2}$$

$$u^2 + v^2 =$$

$$\frac{1}{4} \to \left(\frac{x^2 + \frac{17}{4} * x + y^2 + 1}{(x + 4)^2 + y^2}\right)^2 + \left(\frac{\frac{15}{4} * y}{(x + 4)^2 + y^2}\right)^2 =$$

$$1 - \frac{15\,(17 + 8\,x)}{16\,((4 + x)^2 + y^2)} = \frac{1}{4}$$

$$\to (4 + x)^2 + y^2 = \frac{25}{4}$$ A circle

(b)

$$1 - \frac{15\,(17 + 8\,x)}{16\,((4 + x)^2 + y^2)} =$$

$$\frac{1}{16}\; 0 \to x^2 + y^2 = 1$$ A circle

(7)

$$z = x + y\,i, \quad \bar{z} = x - y\,i$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial x} * \frac{\partial x}{\partial z} = \frac{\partial}{\partial x} * 1 = \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial z} = \frac{\partial}{\partial y} * \frac{\partial y}{\partial z} = \frac{\partial}{\partial y} * \left(\frac{1}{i}\right) = -i\,\frac{\partial}{\partial y}$$

$$\frac{\partial}{\partial \bar{z}} = \frac{\partial}{\partial x} * \frac{\partial x}{\partial \bar{z}} = \frac{\partial}{\partial x} * 1 = \frac{\partial}{\partial x}$$

$$\frac{\partial}{\partial \bar{z}} = \frac{\partial}{\partial y} * \frac{\partial y}{\partial \bar{z}} = \frac{\partial}{\partial y} * \left(\frac{-1}{i}\right) = i\,\frac{\partial}{\partial y}$$

(8)

$$F = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$F^T = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

$$U^2 = F^T F = \begin{pmatrix} a^2 + c^2 & a\,b + c\,d \\ a\,b + c\,d & b^2 + d^2 \end{pmatrix}$$

$$\text{Eigensystem}[F^T F] = \Big\{\Big\{\frac{1}{2}\left(a^2 + b^2 + c^2 + d^2 - \sqrt{(-a^2 - b^2 - c^2 - d^2)^2 - 4\,(b^2 c^2 - 2\,a\,b\,c\,d + a^2 d^2)}\right),$$
$$\frac{1}{2}\left(a^2 + b^2 + c^2 + d^2 + \sqrt{(-a^2 - b^2 - c^2 - d^2)^2 - 4\,(b^2 c^2 - 2\,a\,b\,c\,d + a^2 d^2)}\right)\Big\},$$
$$\Big\{\Big\{-\frac{-a^2 + b^2 - c^2 + d^2 + \sqrt{(-a^2 - b^2 - c^2 - d^2)^2 - 4\,(b^2 c^2 - 2\,a\,b\,c\,d + a^2 d^2)}}{2\,(a\,b + c\,d)}, 1\Big\},$$
$$\Big\{-\frac{-a^2 + b^2 - c^2 + d^2 - \sqrt{(-a^2 - b^2 - c^2 - d^2)^2 - 4\,(b^2 c^2 - 2\,a\,b\,c\,d + a^2 d^2)}}{2\,(a\,b + c\,d)}, 1\Big\}\Big\}\Big\}$$

$$\Psi = \begin{pmatrix} \dfrac{a^2 - b^2 + c^2 - d^2 - \sqrt{((b+c)^2 + (a-d)^2)\,((b-c)^2 + (a+d)^2)}}{2\,(a\,b + c\,d)\sqrt{-\dfrac{\sqrt{((b+c)^2 + (a-d)^2)\,((b-c)^2 + (a+d)^2)}}{a\,b + c\,d}}} & \dfrac{1}{\sqrt{-\dfrac{\sqrt{((b+c)^2 + (a-d)^2)\,((b-c)^2 + (a+d)^2)}}{a\,b + c\,d}}} \\ \dfrac{a^2 - b^2 + c^2 - d^2 + \sqrt{((b+c)^2 + (a-d)^2)\,((b-c)^2 + (a+d)^2)}}{2\,(a\,b + c\,d)\sqrt{-\dfrac{\sqrt{((b+c)^2 + (a-d)^2)\,((b-c)^2 + (a+d)^2)}}{a\,b + c\,d}}} & \dfrac{1}{\sqrt{-\dfrac{\sqrt{((b+c)^2 + (a-d)^2)\,((b-c)^2 + (a+d)^2)}}{a\,b + c\,d}}} \end{pmatrix}$$

$$\sum =$$

$$\left(\begin{array}{cc} \sqrt{\frac{1}{2}\left(a^2 + b^2 + c^2 + d^2 - \sqrt{(-a^2 - b^2 - c^2 - d^2)^2 - 4\,(b^2 c^2 - 2\,a\,b\,c\,d + a^2 d^2)}\right)} & \\ 0 & \sqrt{\frac{1}{2}\left(a^2 + b^2 + c^2 + \right.} \end{array} \right.$$

$$\Psi^T = \begin{pmatrix} \frac{a^2-b^2+c^2-d^2-\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}{2(a\,b+c\,d)\sqrt{-\frac{\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}{a\,b+c\,d}}} & \frac{a^2-b^2+c^2-d^2+\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}{2(a\,b+c\,d)\sqrt{-\frac{\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}{a\,b+c\,d}}} \\ \frac{1}{\sqrt{-\frac{\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}{a\,b+c\,d}}} & \frac{1}{\sqrt{-\frac{\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}{a\,b+c\,d}}} \end{pmatrix}$$

$$U = \Psi \sum \Psi^T = \begin{pmatrix} \frac{\sqrt{a^2+b^2+c^2+d^2-\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}\left(-a^2+b^2-c^2+d^2+\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}\right)+\left(a^2- \ldots\right.}{2\sqrt{2}\sqrt{((b+c)^2+(a-d)^2)((\ldots}} \\ \frac{\sqrt{2}\,(a\,b+c\,d)}{\sqrt{a^2+b^2+c^2+d^2-\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}+\sqrt{a^2 \ldots}} \end{pmatrix}$$

$$R = FU^{-1} = \begin{pmatrix} \frac{\ldots}{d\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}\sqrt{a^2+b^2+c^2+d^2-\sqrt{((b+c)^2+(a-d)^2)((b \ldots}}} \\ b\sqrt{a^2+b^2+c^2+d \ldots} \end{pmatrix}$$

$$V^2 = FF^T = \begin{pmatrix} a^2+b^2 & a\,c+b\,d \\ a\,c+b\,d & c^2+d^2 \end{pmatrix}$$

$$\text{Eigensystem}[FF^T] = \Bigg\{\Bigg\{\frac{1}{2}\left(a^2+b^2+c^2+d^2-\sqrt{(-a^2-b^2-c^2-d^2)^2-4(b^2c^2-2\,a\,b\,c\,d+a^2d^2)}\right),$$
$$\frac{1}{2}\left(a^2+b^2+c^2+d^2+\sqrt{(-a^2-b^2-c^2-d^2)^2-4(b^2c^2-2\,a\,b\,c\,d+a^2d^2)}\right)\Bigg\},$$
$$\Bigg\{\Bigg\{-\frac{-a^2-b^2+c^2+d^2+\sqrt{(-a^2-b^2-c^2-d^2)^2-4(b^2c^2-2\,a\,b\,c\,d+a^2d^2)}}{2(a\,c+b\,d)},1\Bigg\},$$
$$\Bigg\{-\frac{-a^2-b^2+c^2+d^2-\sqrt{(-a^2-b^2-c^2-d^2)^2-4(b^2c^2-2\,a\,b\,c\,d+a^2d^2)}}{2(a\,c+b\,d)},1\Bigg\}\Bigg\}\Bigg\}$$

$$\Phi = \begin{pmatrix} \frac{-a^2-b^2+c^2+d^2+\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}{2\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}} & -\frac{a\,c+b\,d}{\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}} \\ \frac{-a^2-b^2+c^2+d^2-\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}{2\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}} & -\frac{a\,c+b\,d}{\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}} \end{pmatrix}$$

$$\sum =$$

$$\begin{pmatrix} \sqrt{\frac{1}{2}\left(a^2+b^2+c^2+d^2-\sqrt{(-a^2-b^2-c^2-d^2)^2-4\,(b^2c^2-2\,a\,b\,c\,d+a^2d^2)}\right)} & \\ 0 & \sqrt{\frac{1}{2}\left(a^2+b^2+c^2+\right.} \end{pmatrix}$$

$$\Phi^T = \begin{pmatrix} \frac{-a^2-b^2+c^2+d^2+\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}{2\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}} & \frac{-a^2-b^2+c^2+d^2-\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}{2\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}} \\ -\frac{a\,c+b\,d}{\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}} & -\frac{a\,c+b\,d}{\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}} \end{pmatrix}$$

$$V = \Phi \sum \Phi^T = \begin{pmatrix} \frac{\sqrt{a^2+b^2+c^2+d^2-\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}\left(-a^2-b^2+c^2+d^2+\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}\right)+\left(a^2+b\right.}{2\sqrt{2}\sqrt{((b+c)^2+(a-d)^2)((\ }} \\ \frac{\sqrt{2}\,(a\,c+b\,d)}{\sqrt{a^2+b^2+c^2+d^2-\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}+\sqrt{a^2+}} \end{pmatrix}$$

$$R = V^{-1}\,F = \begin{pmatrix} \frac{}{d\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}\sqrt{a^2+b^2+c^2+d^2-\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}+(2\,a\,b\,c-a^2\,d+d\,(b^2+c^2+} \\ \frac{b\sqrt{a^2+b^2+c^2+d^2-\sqrt{((b+c)^2+(a-d)^2)((b-c)^2+(a+d)^2)}}+\frac{(a^2\,b+2\,a\,c\,d+b}{}}{} \end{pmatrix}$$

$$\Phi = \begin{pmatrix} \mathrm{Cos}[\phi] & -\mathrm{Sin}[\phi] \\ \mathrm{Sin}[\phi] & \mathrm{Cos}[\phi] \end{pmatrix}$$

$$\Psi^T = \begin{pmatrix} \mathrm{Cos}[\varphi] & -\mathrm{Sin}[\varphi] \\ \mathrm{Sin}[\varphi] & \mathrm{Cos}[\varphi] \end{pmatrix}$$

$$f(z) = \frac{1}{2}\left(\sqrt{\frac{1}{2}\left(a+d-\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)}+\sqrt{\frac{1}{2}\left(a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)}\right)z + \frac{1}{2}\left(\sqrt{\frac{1}{2}\left(a+d-\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)}-\sqrt{\frac{1}{2}\left(a+d+\sqrt{a^2+4\,b\,c-2\,a\,d+d^2}\right)}\right)\bar{z}$$

$$F = e^{i*\phi}\,f\left(e^{i*\varphi}\,z\right)$$