海洋大學河海工程學系2002 彈性力學期中考(OPEN BOOK)

1. Given a geometry description of the deformation

$$x_1 = X_1 - \tau X_2 X_3 \tag{1}$$
$$x_2 = X_2 + \tau X_1 X_3 \tag{2}$$
$$x_3 = X_3 \tag{3}$$

where τ is a constant. Please answer the following questions.
(a). Is this plane deformation only ? (5 %)
(b). Find the deformation gradient F. (5 %)
(c). Find the right Cauchy-Green strain tensor C. (5 %)
(d). Find the Lagrangian strain tensor L. (5 %)
(e). Is the deformation isochoric, i.e., no volume change. (5 %)

2. Conventionally, the deformation gradient F was decomposed by

$$F = RU \text{ or } F = VR \tag{4}$$

where $\mathbf{dx} = F\mathbf{dX}$. In our course, we present a new concept of singular value decomposition (SVD),

$$F = \Phi\Sigma\Psi^T \tag{5}$$

Please determine the relation of R and Φ and Ψ. (5 %) Write down the procedures to derive Φ, Ψ and Σ. (5 %) Also, if we introduce two new vectors,

$$\mathbf{dy} = \Phi^T\mathbf{dx} \text{ and } \mathbf{dY} = \Psi^T\mathbf{dX} \tag{6}$$

please determine the formula between $\mathbf{dy}$ and $\mathbf{dY}$. (10 %) Please explain the physical, geometrical and numerical meanings for this transformation. (10 %)

3. Can the following functions be the Airy stress function? In another words, are the following functions biharmonic ? (20 %) (1). $f(x,y)$, (2). $xf(x,y)$, (3). $yf(x,y)$, (4). $(x^2+y^2)f(x,y)$, (5). r^2, (6). $ln(r)$, (7). $sin(\theta)/r$, (8). $r^2cos(\theta)$, where $f(x,y)$ is a harmonic function, (x,y) and (r,θ) are the Cartesian and polar coordinates, respectively.

4. Write down the equilibrium equation and compatibility condition for 2-D elasticity. (10 %)

5. Given a stress matrix at a point

$$\sigma_{ij} = \begin{bmatrix} \sigma_{11} & 2 & 1 \\ 2 & 0 & 2 \\ 1 & 2 & 0 \end{bmatrix}$$

Choose σ_{11} (5 %) so that there will be a traction-free plane and determine the normal vector of the plane. (10 %)