

Applications of ODE

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- mixing of solution

$$\frac{dQ}{dt} = 10 - \frac{Q}{20}, Q(0) = 100$$

If $Q(t_f) = 150$, find t_f . (nonhomogeneous standard first order ODE)

- leakage through an orifice

$$\pi(2yR - y^2)dy = -\pi r^2 \sqrt{2gy} dt, y(0) = R$$

If $y(t_f) = 0$, find t_f . (separable)

- dissolving of a solid in a liquid

$$\frac{dQ}{dt} = \frac{k}{120}(20 - Q)(40 - Q), Q(0) = 0$$

If $Q(12) = 4$, find $k = \frac{1}{2}\ln(9/8)$. (separable, fraction type)

- heat loss from a pipe

$$dT = \frac{Q}{2\pi k} \frac{dr}{r}, T(r_0) = T_0$$

(separable)

- laminar fluid flow

$$\frac{d}{dr}(r \frac{dv}{dr}) = \frac{-r(p_0 - p_1)}{nl}, v(0) = finite, v(a) = 0$$

(separable)

- orbital motion

$$\frac{dv}{dr} - \frac{2}{r}v = \frac{r}{r_c}(r_c - r)v^{-1}$$

(Bernoulli form)

- exponential population growth

$$\frac{dN}{dt} = \frac{r}{100}N, N(0) = N_0$$

(separable)

- A finite population model

$$\frac{dN}{dt} = (a - bN)N, N(0) = N_0$$

(separable)