

Bernoulli equation : nonlinear form

$$\dot{y}(x) + a(x)y(x) = f(x)y^n(x)$$

Change of variables: nonlinear transformation

$$z(x) = y^{1-n}(x)$$

Linear ODE : standard form

$$\dot{z}(x) + (1-n)a(x)z(x) = (1-n)f(x)$$

Special case : nonlinear form

$$\dot{y}(x) + a(x)y(x) = f(x)y \ln(y)$$

Change of variables: nonlinear transformation

$$z(x) = \ln(y)$$

Linear ODE : standard form

$$\dot{z}(x) + a(x)z(x) = f(x)$$

Ricatti equation : nonlinear form for control theory

$$\dot{y}(x) = P(x)y^2 + Q(x)y + R(x)$$

$$R(x) = 0 \rightarrow \text{Bernoulli equation}$$

$$R(x) \neq 0 \rightarrow \text{how to solve}$$

Clairaut equation :

$$y = xy'(x) + f(y')$$

Examples :

$$3xy'(x) + y(x) = x^2y^4(x)$$

$$\dot{y}(x) + y(x) = xy^3(x), y(0) = 1$$

$$\dot{y}(x) + y(x)/x = y^2(x)\ln(x), y(1) = 1$$

Determine  $a(x)$ ,  $f(x)$  and  $n$ .

Determine the solution.