Second order nonlinear ODE:

$$y'' = c\sqrt{1 + y'^2}$$

Boundary conditions:

$$y(0) = 0, y'(0) = 0$$

Reducible from second order to first order

$$v = y'$$

$$v\frac{dv}{dy} = c\sqrt{1 + v^2}$$

$$\sqrt{v^2 + 1} = cy + 1$$

Solve first order ODE again:

$$y'^2 = (cy + 1)^2 - 1$$

Setting new variable u

$$u = cy + 1$$

First order ODE change to

$$\frac{du}{\sqrt{u^2 - 1}} = c\ dx$$

$$cosh^{-1}(u) = cx + k$$

$$cosh(cx + k) = u = cy + 1$$

$$k = 0, \ since \ y(0) = 0$$

Final solution:

$$y(x) = \frac{1}{c}\{cosh(cx) - 1\}$$