# 降階法一二階幽雨雨個一階 

Second order nonlinear ODE：

$$
y^{\prime \prime}=c \sqrt{1+y^{\prime 2}}
$$

Boundary conditions：

$$
y(0)=0, y^{\prime}(0)=0
$$

Reducible from second order to first order

$$
\begin{aligned}
& v=y^{\prime} \\
& v \frac{d v}{d y}=c \sqrt{1+v^{2}} \\
& \sqrt{v^{2}+1}=c y+1
\end{aligned}
$$

Solve first order ODE again：

$$
y^{\prime 2}=(c y+1)^{2}-1
$$

Setting new variable $u$

$$
u=c y+1
$$

First order ODE change to

$$
\begin{aligned}
& \frac{d u}{\sqrt{u^{2}-1}}=c d x \\
& \cosh ^{-1}(u)=c x+k \\
& \cosh (c x+k)=u=c y+1 \\
& k=0, \text { since } y(0)=0
\end{aligned}
$$

Final solution：

$$
y(x)=\frac{1}{c}\{\cosh (c x)-1\}
$$

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