正合方程式

海大河工系 陳正宗

Geometry meaning : surface description

$$z = g(x, y)$$

Contour line : z =constant

$$c = g(x, y)$$

Steep descent or no descent :

$$\frac{\Delta z}{\Delta \epsilon} = 0 = \frac{\partial g}{\partial x}dx + \frac{\partial g}{\partial y}dy = \nabla g \cdot (dx, dy)$$

Corresponding ODE :

$$\frac{dy}{dx} = \frac{-\frac{\partial g}{\partial x}}{\frac{\partial g}{\partial y}}$$

Corresponding ODE :

$$Mdx + Ndy = 0$$

Corresponding pair :

$$M = \frac{\partial g}{\partial x}, N = \frac{\partial g}{\partial y}$$

Test criteria :

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Examples :

$$(1 - sin(x)tan(y))dx + (cos(x)sec^2(y))dy = 0$$

$$M = 1 - sin(x)tan(y), N = cos(x)sec^2(y)$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Find $g(x,y)$:

$$\frac{\partial g}{\partial x} = 1 - sin(x)tan(y)$$

$$g(x,y) = x + cos(x)tan(y) + P(y)$$

$$\frac{\partial g}{\partial y} = cos(x)sec^2(y) + P'(y) = cos(x)sec^2(y)$$

$$P(y) = constant$$

$$g(x,y) = x + cos(x)tan(y) + c$$

where c is determined by initial or boundary conditions.