線性微分方程式-解的存在、唯一與合理性？

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Definition: normal ODE

A linear differential equation is normal on an interval I if and only if its coefficient functions and its nonhomogeneous term, if it has one, are continuous and the value of leading coefficient is never zero on I.

Existence theorem:

If x_0 is contained in an interval over which the ODE is normal, and if $k_0, k_1, \cdots, k_{n-1}$ are arbitrary real numbers, then there exists exactly one solution $y(x)$ on I such that $y(x_0) = k_0, y'(x_0) = k_1, \cdots, y^{(n-1)}(x_0) = k_{n-1}$.

Solution families:

$y_1(x)$ satisfy n^{th} homogeneous ODE
$y_2(x)$ satisfy n^{th} homogeneous ODE
$\cdots$ satisfy n^{th} homogeneous ODE
$y_n(x)$ satisfy n^{th} homogeneous ODE
$y_p(x)$ satisfy n^{th} nonhomogeneous ODE

Then, the general solution can be written as

$$y(x) = c_1 y_1(x) + c_2 y_2(x) + \cdots + c_n y_n(x) + y_p(x)$$

$y_1, y_2 \cdots y_n$ are called complementary solutions.

y_p is called a particular solution.

$y(x)$ is called a general solution.

$c_1, c_2 \cdots c_n$ are determined by the n initial conditions.