微分方程的解析方法

海大河海系 陳正宗

Problem statement

$$\frac{dy}{dt} = \alpha y, y(0) = y_0$$

Method 1: by view

$$y(t) = y_0 e^{\alpha t}$$

Method 2:separation of variables

$$\frac{dy}{dt} = \alpha y, y(0) = y_0$$

$$\int \frac{dy}{y} = \int \alpha dt$$

$$ln(y) = \alpha t + c$$

$$y(t) = e^{\alpha t + c} = K e^{\alpha t}$$

$$y(0) = y_0 \ \rightarrow \ y(t) = e^{\alpha t + c} = y_0 e^{\alpha t}$$

Method 3: series solution

$$y(t) = y_0 + y_1 t + y_2 t^2 + \cdots + y_n t^n + \cdots$$

$$\dot{y}(t) = y_1 + 2y_2 t + \cdots + n y_n t^{n-1} + \cdots$$

$$\alpha y(t) = \alpha y_0 + \alpha y_1 t + \cdots + \alpha y_n t^n + \cdots$$

Comparing the coefficients, we have

$$y_n = \frac{1}{n!} \alpha^n y_0$$

$$y(t) = y_0 e^{\alpha t}$$

Method 4: succesive iteration method

$$\int_0^t dy(t) = \alpha \int_0^t y(t) dt \ \rightarrow y(t) = y(0) + \alpha \int_0^t y(t) dt$$

$$y_1(t) = y_0$$

$$y_2(t) = y_0 + \alpha y_0 t$$

$$\cdots = \cdots$$

$$y_n(t) = y_0 + y_0 \alpha t + \frac{1}{2!} y_0 (\alpha t)^2 + \frac{1}{3!} y_0 (\alpha t)^3 + \cdots$$

$$y(t) = y_0 e^{\alpha t}$$