人口成長模式

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population explosion model: growth rate is proportional to current population

$$\dot{P}(t) = \alpha P(t), \alpha > 0$$

population decay model: growth rate is proportional to current population

$$\dot{P}(t) = \alpha P(t), \alpha < 0$$

population saturated model: growth rate is proportional to quadratic form of population

$$\dot{P}(t) = P(t)(\beta - \delta P)$$

Three cases of initial conditions:

Case 1: unreasonable

$$P(0) < 0$$

Case 2: grow to be saturated

$$0 < P(0) < \frac{\beta}{\delta}$$

Case 3: decay to be saturated

$$P(0) > \frac{\beta}{\delta}$$

general solution is :

$$P(t) = \frac{\beta}{\delta + [\frac{\beta}{P(0)} - \delta]e^{-\beta t}}$$

Asymptotic population= $\frac{\beta}{\delta}$.

Use Mathematica to plot the curves.

Existence and uniqueness for a solution(if f, f_y are differetial)

$$dy/dx = f(x, y), y(x_0) = y_0 \rightarrow \textit{an existent and unique solution}$$