

Homogeneous function with m degrees:

$$f(\lambda x_1, \lambda x_2, \dots, \lambda x_m) = \lambda^m f(x_1, x_2, \dots, x_m)$$

Homogeneous differential equation $f(x) = 0$:

$$a_2 y''(x) + a_1 y'(x) + a_0 y(x) = f(x)$$

Nonhomogeneous differential equation $f(x) \neq 0$:

$$a_2 y''(x) + a_1 y'(x) + a_0 y(x) = f(x)$$

Homogeneous function:

$$M(x, y) = M(\lambda x, \lambda y)$$

$$N(x, y) = N(\lambda x, \lambda y)$$

Transformation from nonseparable problem to separable problem:

$$\frac{dy}{dx} = \frac{-M(x, y)}{N(x, y)}$$

$$\frac{dy}{dx} = \frac{-M(1, y/x)}{N(1, y/x)} = F(y/x)$$

Setting $y = ux$, we have

$$\frac{dy}{dx} = u + x \frac{du}{dx}$$

Then the nonexact form can be reduced to

$$xdu = (F(u) - u)dx$$

$$\frac{1}{x}dx = \frac{1}{(F(u) - u)}du$$