點疊代 $x_0, x_1, x_2 \cdots$

$$x_{n+1} = g(x_n)$$

收斂準則: $\mid g'(x_0) \mid < 1$, 亦即

$$(1 - f'(x_0)) < 1, \rightarrow f'(x_0) > 0$$

Examples: $x^2 - 3 = 0$

$x_0 = 2.0$ initial guess:

Case 1: $x = \frac{1}{2}(x + \frac{3}{x})$ (OK)

Case 2: $x = x^2 + x - 3$ (NG)

Case 3: $x = \frac{3}{x}$ (NG)

函數疊代 $x_0(t), x_1(t), x_2(t), \cdots$

$$x_{n+1}(t) = \mathcal{F}\{x_n(t)\}$$

如

$$\dot{y}(t) = ay(t)$$

疊代式

$$y(t) = y(0) + \int_0^t ay(\tau)d\tau$$

土壤阻尼動力方程:

$$-kx(t) = m\ddot{x}(t) - \frac{k\eta}{\pi} \int_{-\infty}^{\infty} \frac{x(u)}{(t-u)}du$$

向量疊代 $\tilde{x}_0, \tilde{x}_1(t), \tilde{x}_2(t), \cdots$

$$\tilde{x}_{n+1}(t) = \mathcal{F}\{\tilde{x}_n(t)\}$$

如

$$\frac{d\tilde{x}(t)}{dt} = \mathbf{A}\tilde{x}(t)$$

疊代式

$$\tilde{x}(t) = \tilde{x}(0) + \int_0^t \mathbf{A}\tilde{x}(\tau)d\tau$$

State: point, function and vector.