# Bernoulli 方程與人口成長模式

海大河海系 陳正宗

1. Transformation of dependent variable(Bernoulli equation) : nonlinear form to linear

   $$\dot{y}(x)+a(x)y(x)=f(x)y^n(x)$$

   Change of dependent variables: nonlinear transformation

   $$z(x)=y^{1-n}(x)$$

   Linear ODE : standard form

   $$\dot{z}(x)+(1-n)a(x)z(x)=(1-n)f(x)$$

   Special case : nonlinear form

   $$\dot{y}(x)+a(x)y(x)=f(x)\,y\,ln(y)$$

   Change of independent variables: nonlinear transformation

   $$z(x)=ln(y)$$

   Linear ODE : standard form

   $$\dot{z}(x)+a(x)=f(x)z$$

2. Example: logistic population model

   Bernoulli ODE :

   $$y'-Ay=By^2$$

   Separable ODE :

   $$y'=Ay-By^2=(y)(A-By)$$

   Change of independent variable:

   $$u=1/y$$

   ODE for $u$ :

   $$u'+Au=B$$

   Solution:

   $$u(x)=\frac{1}{(B/A)+ce^{-Ax}}$$