

Complementary solution

$$y''''(x) - y(x) = 0$$

Sol. : $y_h(x) = e^x, e^{-x}, \cos(x), \sin(x), \cosh(x), \sinh(x)$

Only four functions are independent to match four conditions.

Particular solution

$$y''''(x) - y(x) = 1$$

Sol. : $y_p(x) = -1$

Any particular solution plus a complementary solution is another particular solution.

Example:

$$u''(x) = \sin(x)$$

$$u(x) = a + bx - \sin(x)$$

where a and b are determined by boundary conditions or initial conditions.

Extension to integral equation:

$$u(x) = \left\{ u(s) \frac{dU(s, x)}{ds} - u'(s)U(s, x) \right\} \Big|_0^\pi + \int_0^\pi U(s, x) \sin(s) ds, 0 < x < \pi$$

where

$$U(s, x) = \frac{1}{2} |x - s|$$

Complementary sol.:

$$u(x) = \left\{ u(s) \frac{dU(s, x)}{ds} - u'(s)U(s, x) \right\} \Big|_0^\pi$$

Particular sol.:

$$u(x) = \int_0^\pi U(s, x) \sin(s) ds = -\sin(x) + \pi, 0 < x < \pi$$