

Separation of space and time

$$f(x, y) = g(x)h(y) \iff ff_{xy} = f_x f_y$$

Proof:  $\implies$

$$ff_{xy} = ghg_x h_y$$

$$f_x f_y = g_x h g h_y$$

Proof:  $\impliedby$

For nontrivial case  $f(x, y) \neq 0$ ,

$$ff_{xy} - f_x f_y = 0 \rightarrow \frac{ff_{xy} - f_x f_y}{f^2(x, y)} = 0 = \frac{\partial}{\partial y} \left\{ \frac{f_x}{f} \right\}$$

$$\left\{ \frac{f_x}{f} \right\} = \alpha(x)$$

$$\frac{\partial}{\partial x} (\ln(f(x, y))) = \alpha(x)$$

$$\ln(f(x, y)) = \int \alpha(x) dx + \beta(y)$$

$$f(x, y) = \exp^{\phi(x)} + \exp^{\beta(y)}$$

$$f(x, y) = \Phi(x)\Psi(y)$$

Example: separable

$$f(x, y) = 2x^2 + y - x^2 y + xy - 2x - 2 = (-x^2 + x + 1)(y - 2)$$

$$f_x = 4x - 2xy + y - 2$$

$$f_y = 1 - x^2 + x$$

$$f_{xy} = -2x + 1$$

$$ff_{xy} = f_x f_y$$

Example: not separable

$$f(x, y) = 1 + xy$$

$$f_x = y$$

$$f_y = x$$

$$f_{xy} = 1$$

$$ff_{xy} \neq f_x f_y$$