Surface representation

$$z = z(x, y)$$

Contour line governed by the following ODE

$$\frac{dy}{dx} = \frac{-\frac{\partial z}{\partial x}}{\frac{\partial z}{\partial y}}$$

Steep descent line governed by the following ODE

$$\frac{dy}{dx} = \frac{\frac{\partial z}{\partial y}}{\frac{\partial z}{\partial x}}$$

Ex.1 $z = z(x, y) = \frac{x^2+y^2-1}{2y}$

Contour line

$$\frac{dy}{dx} = \frac{x^2 - y^2 - 1}{-2xy}$$

$$(x-h)^2 + y^2 = h^2 - 1$$

Steep descent line

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2 - 1}$$

$$x^2 + y^2 - 1 = 2ky$$

Ex.2 $z = z(x, y) = x^2 + y^2$

Contour line

$$\frac{dy}{dx} = \frac{-x}{y}$$

$$x^2 + y^2 = h^2$$

Steep descent line

$$\frac{dy}{dx} = \frac{y}{x}$$

$$y = kx$$