1. Transformation of dependent variable(Bernoulli equation) : nonlinear form to linear

    $$\dot{y}(x) + a(x)y(x) = f(x)y^n(x)$$

    Change of dependent variables: nonlinear transformation

    $$z(x) = y^{1-n}(x)$$

    Linear ODE : standard form

    $$\dot{z}(x) + (1-n)a(x)z(x) = (1-n)f(x)$$

    Special case : nonlinear form

    $$\dot{y}(x) + a(x)y(x) = f(x)\, y\, ln(y)$$

    Change of independent variables: nonlinear transformation

    $$z(x) = ln(y)$$

    Linear ODE : standard form

    $$\dot{z}(x) + a(x) = f(x)z$$

2. Transformation of independent variable(Cauchy-Euler equation): variable coef. ODE to const. coef. ODE

    Variable coefficient ODE :

    $$t^2\ddot{y}(t) + at\dot{y}(t) + by(t) = 0$$

    Change of independent variable:

    $$t = e^x, x = ln(t)$$

    Constant coefficient ODE :

    $$Y"(x) + (a-1)Y'(x) + bY(x) = 0$$

3. Integral transformation of independent variable(Cauchy-Euler equation): diff. operator to algebraic operator

    $$F(s) = \int_0^\infty y(t)e^{-st}dt$$