求解微分方程的轉換方法

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1. Transformation of dependent variable: nonseparable to separable

   Transformation from nonseparable problem to separable problem:

   If $M(x,y)$ and $N(x,y)$ are homogeneous with the same degrees, we have

   $$\frac{dy}{dx} = \frac{-M(x,y)}{N(x,y)}$$

   $$\frac{dy}{dx} = \frac{-M(1,y/x)}{N(1,y/x)} = F(y/x)$$

   Setting $y = ux$, we have

   $$\frac{dy}{dx} = u + x\frac{du}{dx}$$

   Then the nonexact form can be reduced to

   $$xdu = (F(u) - u)dx$$

   $$\frac{1}{x}dx = \frac{1}{(F(u) - u)}du$$

   $(x, y(x)) \rightarrow (x, u(x))$ where $u(x) = y(x)/x$.

2. Transformation of independent variable(Cauchy-Euler equation): variable coef. ODE to const. coef. ODE

   Variable coefficient ODE :

   $$t^2\ddot{y}(t) + at\dot{y}(t) + by(t) = 0$$

   Change of independent variable:

   $$t = e^x, x = ln(t)$$

   Constant coefficient ODE :

   $$Y"(x) + (a-1)Y'(x) + bY(x) = 0$$

   $(t, y(t)) \rightarrow (x, Y(x))$ where $Y(x) = y(t)$.