國立臺灣海洋大學河海工程學系 2001 工程數學 (三) 期中考

1. Given a function $f(x) = \begin{cases} |x| & -1 < x < 1, \\ 0 & otherwise. \end{cases}$

(a) Express the $f(x)$ in terms of Lengendre polynomials

$$f(x) \approx c_0P_0(x)+c_1P_1(x)+c_2P_2(x)+c_3P_3(x)+c_4P_4(x),$$

where

$$P_0(x) = 1,$$
$$P_1(x) = x,$$
$$P_2(x) = \frac{1}{2}(3x^2-1),$$
$$P_3(x) = \frac{1}{2}(5x^3-3x),$$
$$P_4(x) = \frac{1}{8}(35x^4-30x^2+3)$$

Please determine the coefficients of c_0, c_1, c_2, c_3 and c_4. (10 %) Use the Lengendre polynomials to express the function for $f(x) = 5x^3$. (5 %) Find the particular solution, $y_p(x)$, for (10 %)

$$(1-x^2)y_p''(x) - 2xy_p'(x) = 5x^3$$

2. Find the indicial equation for (10 %)

$$x(x-1)y''(x)+3xy'(x)+y(x) = 0$$

3. If $sin(\sqrt{x})$ is one of the complementary solutions, find another one for (10 %)

$$4xy''(x)+2y'(x)+y(x) = 0$$

4. Explain the following items: (30 %)
(1). Lengendre polynomials,
(2). Bessel function,
(3). regular point
(4). Chebyshev polynomial,
(5). Euler-Cauchyequation,
(6). Hermitian matrix.

5. By changing $x = cos(\theta)$ and $y(x) = y(cos(\theta)) = \bar{y}(\theta)$ for

$$(1-x^2)y''(x) - xy'(x)+m^2y(x) = 0,$$

find the ODE for $\bar{y}(\theta)$. (10 %) Also, derive the orthogonal relation for the Chebyshev polynomial. (10%)

6. Find the eigenvalues and eigenfunctions for (10 %)

$$y''(x) = \lambda y(x), y(0) = y(\pi) = 0.$$