Bessel and modified Bessel functions

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Bessel functions:

$$x^2 y''(x) + xy'(x) + (x^2 - n^2)y(x) = 0$$

Modified Bessel functions:

$$x^2 y''(x) + xy'(x) + (-x^2 - n^2)y(x) = 0$$

Transformation:

$$y(x) = Y(ix) = Y(X)$$
$$ix = X$$

Exponential function:

$$y''(x) - n^2 y(x) = 0$$

Harmonic function:

$$y''(x) + n^2 y(x) = 0$$

Transformation:

$$y(x) = Y(ix) = Y(X)$$
$$ix = X$$

Scaled Bessel functions:

$$x^2 y''(x) + xy'(x) + (\lambda^2 x^2 - n^2)y(x) = 0$$

Transformation:

$$x, y(x) \ \rightarrow \ \bar{x}, \bar{y}(\bar{x})$$

where $\bar{x} = \lambda x$.

$$\bar{x}^2 \bar{y}''(x) + \bar{x}\bar{y}'(\bar{x}) + (\bar{x}^2 - n^2)\bar{y}(\bar{x}) = 0$$

Solution is $J_n(\lambda x)$.