系級:______________ 姓名:______________ 學號:______________

Sturm-Liouvilli System

Function	ODE	$p(x)$	$q(x)$	$\rho(x)$	λ
Legendre polynomial	$(1-x^2)y''-2xy'+N(N+1)y=0$	$1-x^2$	0	1	$N(N+1)$
Bessel function	$x^2y''+xy'+(x^2-\nu^2)y=0$	x	x	$\frac{1}{x}$	$-\nu^2$
Simple Harmonic Motion function	$y''+\omega^2y=0$	1	0	1	ω^2
Hermite function	$y''-2xy'+2\alpha y=0$	e^{-x^2}	0	e^{-x^2}	2α
Laguerre function	$xy''+(1-x)y'+ny=0$	xe^{-x}	0	e^{-x}	n
Chebyshev polynomial	$(1-x^2)y''-xy'+n^2y=0$	$\frac{1}{\sqrt{1-x^2}}$	0	$\frac{1}{\sqrt{1-x^2}}$	n^2

Strum-Liouville ODE

$$(py')'+qy=-\lambda\rho y$$

$$py''+qy'+\lambda\rho y=0$$

$$(Fp)y''+(Fr)y'+Fqy+F\lambda\rho y=0$$

$$Fp=\overline{p},\ Fr=\overline{r},\ Fq=\overline{q},\ F\lambda\rho=\lambda\overline{\rho}$$

$$\overline{p}y''+\overline{r}y'+\overline{q}y+\lambda\overline{\rho}y=0$$

$$\overline{p}'=\overline{r},\ (Fp)'=(Fr),\ F'p+Fp'=Fr$$

$$pF'+(p'-r)F=0$$

$$F=e^{-\int\frac{p'-r}{p}dx}$$