

1. By defining the Sturm-Liouville operator,

$$\mathcal{L}\{y(x)\} = \left(p(x)\frac{dy(x)}{dx}\right)' + r(x)y(x)$$

find the adjoint operator \mathcal{L}^* such that

$$\int_a^b u(x)\mathcal{L}\{v(x)\}dx = \int_a^b v(x)\mathcal{L}^*\{u(x)\}dx + J(u(x), v(x))\Big|_{x=a}^{x=b}$$

Determine \mathcal{L}^* and $J(u(x), v(x))$.

2. In which condition, can we have $J(u(x), v(x)) = 0$?