系級:_____ 姓名:_____ 學號:_____

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1.

$\mathcal{L} = \dfrac{d^2}{dx^2}\,,\ y(0) = y(1) = 0$

$\mathcal{L}\{y_n\} = \lambda_n y_n$

$y_n = a\cos(\sqrt{\lambda_n}x) + b\sin(\sqrt{\lambda_n}x)$

$\therefore \lambda_n = -n^2\pi^2$

$y_n = \sin(n\pi x)$

2.

$\mathcal{L}\{y(x)\} = f(x) = \sum c_n y_n(x)$

$c_n = \dfrac{\int_a^b f(x)y_n(x)dx}{\lambda_n \int_a^b y_n^2 dx}$

$\displaystyle\int_a^b y_n^2 dx = \int_0^1 (\sin(n\pi x))^2 dx = \frac{1}{2}$

$y_n = \dfrac{\sin(n\pi x)}{\sqrt{\frac{1}{2}}} = \sqrt{2}\sin(n\pi x)$

$\displaystyle y(x) = \sum_{n=1}^{\infty} \frac{y_n(x)}{\lambda_n} \int_a^b f(s)y_n(s)ds$

$\displaystyle = \int_a^b (\sum_{n=1}^{\infty} \frac{y_n(x)y_n(s)}{\lambda}) f(s)ds$

$\displaystyle G(x,s) = \sum_{n=1}^{\infty} \frac{y_n(x)y_n(s)}{\lambda_n} = 2\sum_{n=1}^{\infty} \frac{\sin(n\pi x)\sin(n\pi s)}{-n^2\pi^2}$

3.

$f(x) = x$

$\displaystyle y = \int_0^1 (2\sum_{n=1}^{\infty} \frac{\sin(n\pi x)\sin(n\pi s)}{-n^2\pi^2}) s ds$

$\displaystyle = 2\int_0^1 -\frac{s}{\pi^2}(\sin(\pi x)\sin(\pi s) + \frac{\sin(2\pi x)\sin(2\pi s)}{4} + ...) ds$

$\displaystyle = \sum_{n=1}^{\infty} \frac{2(-1)^n \sin(n\pi x)}{(n\pi)^3}$