國立臺灣海洋大學河海工程學系 2002 工程數學 (三) 期末考

1. Explain the following items. (15 %)
(1). Symmetric matrix, (2). Hermitian matrix, (3). Hermitian operator, (4). Green's function and (5). Fundamental solution

2. Determine the eigenvalues and eigenfunctions for $\frac{d^2y_n(x)}{dx^2} = \lambda_n y_n(x)$ subject to $y_n(0) = 0$ and $y'(1) = 0$. (10 %) Solve the Green's function $y(x) = G(x, s)$ in terms of closed form and series form for

$$\frac{d^2y(x)}{dx^2} = \delta(x - s)$$

subject to $y(0) = 0$ and $y'(1) = 0$. (10 %) Solve the particular solution in terms of closed form and series form for

$$\frac{d^2y(x)}{dx^2} = cos(x)$$

subject to $y(0) = 0$ and $y'(1) = 0$. (10 %)

3. Change the two ODEs to the Sturm-Liouville forms of $(py')' + qy = -\lambda\rho y$. (20 %)

$$y''(x) - 2xy'(x) + 2\alpha y(x) = 0$$

$$(1 - x^2)y''(x) - xy'(x) + n^2y(x) = 0$$

Please detemine the functions of p, q, ρ and the eigenvalue of λ.

4. Find the solution

$$(1 - x^2)y''(x) - 2xy'(x) + 14y(x) = 5x^3$$

where the Lengendre polynomials are $P_0(x) = 1, P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2 - 1)$ and $P_3(x) = \frac{1}{2}(5x^3 - 3x)$. First determine c_n such that $5x^3 = \sum_{n=0}^{3} c_n P_n(x)$. (10 %) Then solve the particular solution using eigenfunction expansion. (10 %)

5. By substituting $x = e^t$, find the normalized eigenfunction $y_n(x)$ (10 %) and the eigenvalues λ_n (10 %) for the operator $\mathcal{L}$ defined by

$$\mathcal{L} = x^2y'' + 2xy' + \frac{1}{4}y, \quad \text{subject to} \quad y(1) = y(e) = 0$$

Find the solution for $\mathcal{L}\{y(x)\} = \frac{1}{\sqrt{x}}$ using $y(x) = \sum a_n y_n(x)$. (10 %)

6. Solve the Green's function in terms of closed form for

$$\frac{d^2y(x)}{dx^2} + \pi^2 y(x) = \delta(x - s)$$

subject to $y(0) = y(1)$ and $y'(0) = y'(1)$. (10 %) Solve the particular solution for

$$\frac{d^2y(x)}{dx^2} + \pi^2 y(x) = cos(\pi x). \text{ (10 \%)}$$