國立臺灣海洋大學河海工程學系 2001 工程數學 (三) 期中考 (open book)

1. (a) Express the function $f(x) = x^2 - 1$ in terms of Lengendre polynomials (10 %)

$$P_0(x) = 1,$$

$$P_1(x) = x,$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1),$$

$$P_3(x) = \frac{1}{2}(5x^3 - 3x),$$

$$P_4(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

Find the particular solution, $y_p(x)$, for (15 %)

$$(1 - x^2)y_p''(x) - 2xy_p'(x) = x^2 - 1$$

2. By changing the variable of $\bar{x} = \sqrt{x}$, transform the ODE

$$4x\, y''(x) + 2\, y'(x) + y(x) = 0$$

to the ODE for $\bar{y}(\bar{x})$ and solve $\bar{y}(\bar{x})$ and $y(x)$, where $y(x) = \bar{y}(\bar{x})$. (25 %)

3. Prove that the eigenvalues of Hermitian matrice are real. (25 %)

4. Given $[A]\mathbf{x} = \mathbf{b}$, where

$$[A] = \begin{bmatrix} 1 & -2 \\ 3 & -4 \end{bmatrix},$$

$$\mathbf{b} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}.$$

Find the eigenvalues and eigenvectors of the matrix $[A]$. (10 %) Using the eigenvector expansion, solve $\mathbf{x}$. (15 %)