級數解的猜法

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1. Solve the ODE using series form:

$$x^2y''(x) - 4xy'(x) - 6y(x) = 0,$$

(a). $y(x) = \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$
(b). $y(x) = \sum_{n=0}^{\infty} c_n x^{-n} = c_0 + c_1 \frac{1}{x} + c_2 \frac{1}{x^2} + c_3 \frac{1}{x^3} + \cdots$
(c). $y(x) = \sum_{n=-\infty}^{\infty} c_n x^{-n}$
(d). $y(x) = \sum_{n=0}^{\infty} c_n x^{n+r}$

2. The definitions of the expansion point
(a). regular

$$p(x) = \sum_{n=0}^{\infty} p_n x^n$$

$$q(x) = \sum_{n=0}^{\infty} q_n x^n$$

$$y(x) = \sum_{n=0}^{\infty} c_n x^n$$

(b). regularlly singular

$$xp(x) = \sum_{n=0}^{\infty} l_n x^n$$

$$x^2q(x) = \sum_{n=0}^{\infty} k_n x^n$$

$$y(x) = \sum_{n=0}^{\infty} c_n x^{n+r}$$

(c). irregularly singular

$$y(x) = ?$$