

8:20-10:10, Apr.1, 1995

I. Solve the PDE

$$u_{tt} = \begin{cases} 4u_{xx}, & for \ x < 0, \ t > 0\\ 1u_{xx}, & for \ x > 0, \ t > 0 \end{cases}$$

with initial conditions

$$u(x,0) = 0, u_t(x,0) = 0$$

and u(x,t) is continuous across x = 0, while

$$u_x(0^+, t) - u_x(0^-, t) = a \sin(\omega t)$$

where a, ω are two constants.

II. Reflection and transmission due to lump mass, m, in one medium using diamond rule. Solve the PDE

$$u_{tt} = c_1^2 u_{xx}, \quad for \ x < 0, \ t > 0$$

with initial condition of displacement

$$u(x,0) = \begin{cases} f(x), & \text{for } x > 0\\ 0, & \text{for } x < 0 \end{cases}$$

with initial condition of velocity

$$u_t(x,0) = \begin{cases} 0, & for \ x > 0 \\ 0, & for \ x < 0 \end{cases}$$

u(x,t) is continuous across x = 0,

$$u(0^+, t) = u(0^-, t)$$

Force can be transmitted across x = 0,

$$m\ddot{u}(t) = \rho c_1^2 u_x(0^+, t) - \rho c_1^2 u_x(0^-, t)$$

(1). Determine the ratio of transmission and reflection.