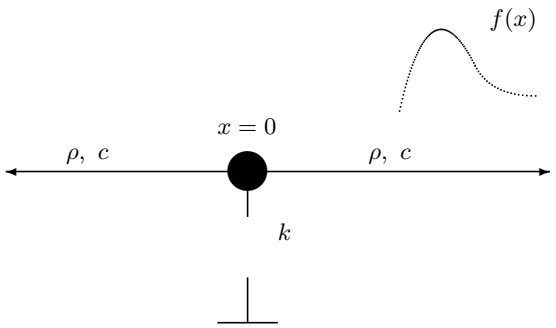
工程數學 (四) - 期末考 (Take home)



(20,12,19,11.5,20,11) (20,11,21,10.5,20,10) (20,10,19,9.5,20,9) (20,9,21,8.5,20,8)

Fig.1 An infinite string with a spring and a lump mass at $x=0$

I. As shown in Fig.1, string reflection and transmission will occur due to a spring at $x=0$ with spring constant, k, and a sphere with lump mass, m, in one medium. Solve the PDE using diamond rule.

$$u_{tt}=c^2u_{xx},\quad for\ \ -\infty<x<\infty,\ \ t>0$$

with initial condition of displacement

$$u(x,0)=\begin{cases} f(x), & for\ \ x>0\\ 0, & for\ \ x<0\end{cases}$$

where the incident wave for $f(x)$ is

$$f(x)=[H(x-5)-H(x-6)](x-5)+[H(x-6)-H(x-7)](-x+7)$$

with initial condition of velocity

$$u_t(x,0)=\begin{cases} 0, & for\ \ x>0\\ 0, & for\ \ x<0\end{cases}$$

$u(x,t)$ is continuous across $x=0$,

$$u(0^+,t)=u(0^-,t)$$

Force can be transmitted across $x=0$,

$$m\ddot{u}(0,t)+ku(0,t)=\rho c^2u_x(0^+,t)-\rho c^2u_x(0^-,t)$$

(1). Determine the solution in each region for $\rho=1,c=1$.
(2). 3-D plot and contour plot for $u(x,t)$.
(3). Plot $u(x,t)$ for fixed x and plot $u(x,t)$ for fixed t.
(4). Determine the ratio of transmission and reflection. Will the barrier system make the wave dispersion ?
(5). Parameteric study for fixed m, changing k. Parameteric study for fixed k, changing m.
(9). Reduce the solution by $m\to 0$ and check the following solution if $k\to 0$

$$r(t)=u(0,t)=\int_0^t\frac{\rho c}{m}e^{-\frac{\rho c}{m}(t-\tau)}f(c\tau)d\tau$$

II. As shown in the course, change the string to beam, repeat the process.

$$\frac{\partial^4u}{\partial x^4}+\frac{\partial^2u}{\partial t^2}=0$$

subected to IC $u(x,0)=0,\dot{u}(x,0)=0$ and Bcs

$$u(0,t)=sin(t),u(1,t)=sin(t)$$