國立海洋大學河海工程學系1998 工程數學 (四) 期中考 OPEN BOOK

1. Consider the Cauchy problem

$$yu_x - xu_y = 0$$

with two Cauchy data

$$u(cos(\theta), sin(\theta)) = g(\theta), -\frac{\pi}{2} < \theta < \frac{\pi}{2}$$

$$u(0, y) = f(y), -1 < y < 1$$

(a). Does the solution exist for any $f(y)$ and $g(\theta)$? (10 %)

(b). If (a) is not correct, then what is the condition of $f(y)$ and $g(\theta)$ which can confirm that there is a solution. Also, solve the solution $u(x, y)$. (10 %)

2. Consider the nonlinear first order PDE as shown below:

$$u_x u_y = 1$$

(a). Find the Monge cone. (10 %)

(b). Given the Cauchy data, $u(s, s) = 2.5s$, find all the solutions. (10 %)

3. Solve the nonlinear first order PDE as shown below (15 %) :

$$u_t + uu_x + 2ux = 0, \;\; 0 < x < 1, 0 < t$$

Given the Cauchy data,

$$u(x, 0) = 4 - x^2, \;\; 0 < x < 1$$

$$u(0, t) = 1, \;\; t > 0$$

4. Explain the characteristic value, charateristic vector, charateristic line(or curve) and charateristic strips. (10 %)

5. Determine the family of circles for Mohr-Columb failure criterion envelope. (10 %)

$$\tau = c + \sigma\, tan(\phi), c \text{ and } \phi \text{ are constants}$$

6. Derive the D'Alembert solution (15 %) :
Governing equation

$$u_{tt} = c_1^2 u_{xx}, \;\; -\infty < x < \infty, \;\; t > 0$$

where c_1 is wave velocity and the Cauchy data are

$$u(x, 0) = \phi(x), \;\; \dot{u}(x, 0) = \psi(x),$$

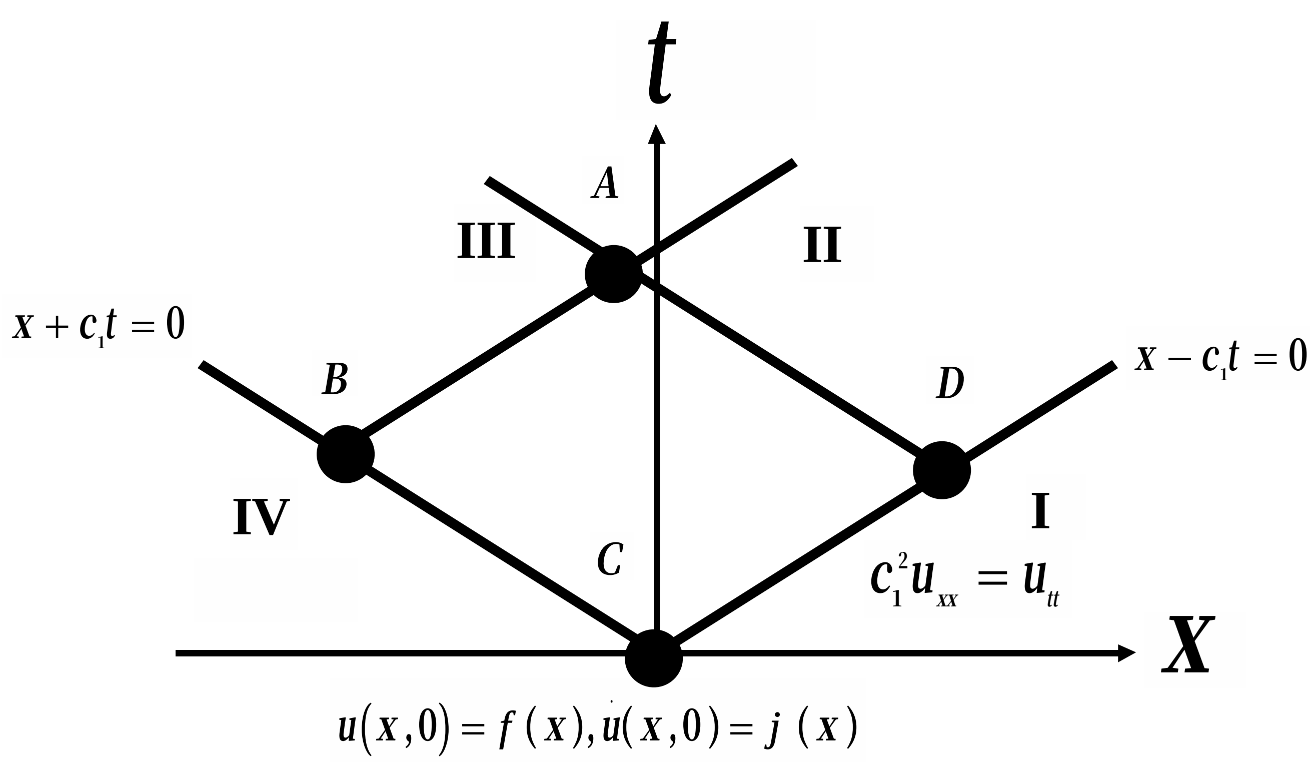
D'Alembert's solution :

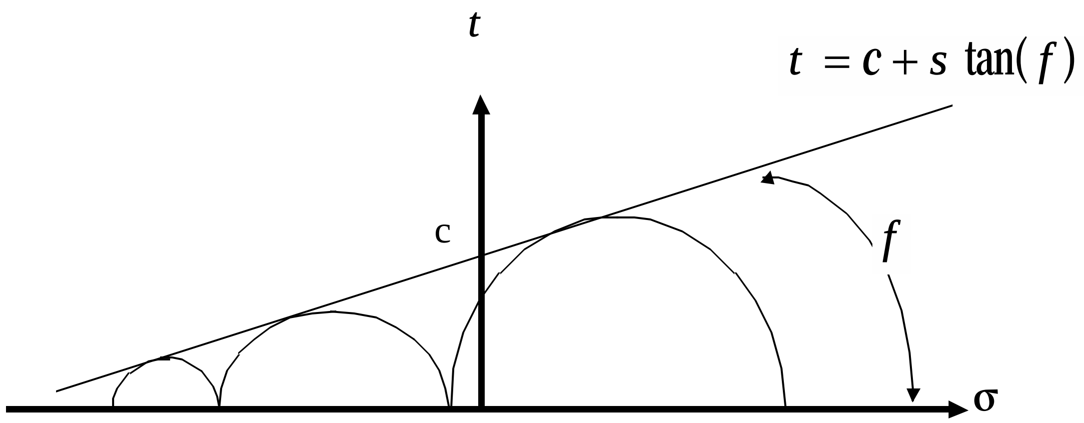
$$u(x, t) = \frac{1}{2}\phi(x + c_1 t) + \frac{1}{2}\phi(x - c_1 t) + \frac{1}{2c_1}\int_{x-c_1 t}^{x+c_1 t} \psi(x)dx$$

7. Prove the diamond rule. (10 %)

$$u_A + u_C = u_B + u_D$$

8. Explain the paradox in the course using the results of **6** and **7**. (10 %)


t
A
III
II
$x + c_1 t = 0$
$x - c_1 t = 0$
B
D
IV
I
C
$c_1^2 u_{xx} = u_{tt}$
X
$u(x,0) = f(x), \dot{u}(x,0) = j(x)$


t
$t = c + s \tan(f)$
c
f
σ