工程數學 (四) - 偏微分方程作業一

I. If the PDE is

$$u_{xx} = u_{yy}$$

Change

$$\xi = x + y$$
$$\eta = x - y$$
$$u_{\xi\eta} = 0$$

II. PDE is

$$2u_x + 3u_y = 4$$

Cauchy data

    Case 1:

$$u(s, s^2) = 5s^2$$

    Case 2:

$$u(2s, 3s) = 5$$

III. Envelope construction
    (1). Clairat's equation
    (2). Mohr-Columb failure criterion
    (3). Monge cone

IV. Wave equation:

$$u_x = u_y$$

Cauchy data:

$$u(s, s) = 2s$$

    PDE:

$$u_x^2 + u_y^2 = 1$$

V. Eikonal equation:
    PDE:

$$u_x^2 + u_y^2 = 1$$

Cauchy data:

$$u(y^2, y) = y$$

VI. General nonlinear first order PDE:
    PDE:

$$F(x, y, u, u_x, u_y) = u - xu_x - \frac{1}{2}u_y^2 + x^2 = 0$$

Cauchy data:

$$u(x, 0) = x^2 - \frac{1}{6}x^4$$