工程數學 (四) - 偏微分方程作業三

I. Please derive the solvability condition for second order PDE

$$a(x,y)u_{xx} + b(x,y)u_{xy} + c(x,y)u_{yy} + du_x + eu_y + qu = p$$

II. Cauchy data

$$u(f(s), g(s)) = h(s)$$
$$u_x(f(s), g(s)) = l(s)$$
$$u_y(f(s), g(s)) = m(s)$$

III. A unique solution can be obtained if

$$det \begin{bmatrix} a(x,y) & b(x,y) & c(x,y) \\ f'(s) & g'(s) & 0 \\ 0 & f'(s) & g'(s) \end{bmatrix} \neq 0$$

IV. A unique solution can be obtained if

$$det \begin{bmatrix} a(x,y) & b(x,y) & c(x,y) & 0 \\ 0 & a(x,y) & b(x,y) & c(x,y) \\ f'^2 & 2f'g' & g'^2 & 0 \\ 0 & f'^2 & 2f'g' & g'^2 \end{bmatrix} \neq 0$$