工程數學 (四) - 偏微分方程作業四

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1. Solve the nonlinear first order PDE as shown below (15 %) :

$$u_t + uu_x + 2ux = 0,\ \ 0 < x < 1, 0 < t$$

Given the Cauchy data,

$$u(x,0) = 4 - x^2,\ \ 0 < x < 1$$

$$u(0,t) = 1,\ \ t > 0$$

Sol:
region I: $u(x,t) = 4 - x^2$
region II: $u(x,t) = c - x^2$, $c \in \mathbf{R}$
region III: $u(x,t) = 1 - x^2$
separate characteristic line:
$t = \frac{1}{4}\ ln\ |\ \frac{x+2}{x-2}\ |$
$t = \frac{1}{2}\ ln\ |\ \frac{x+1}{x-1}\ |$
jump solution can occur only at the characteristic line.