工程數學 (四) - 偏微分方程作業五

1. Explain the paradox in the course Derive the D'Alembert solution (15 %) :
Governing equation

$$u_{tt} = c_1^2 u_{xx}, \ \ -\infty < x < \infty, \ \ t > 0$$

where c_1 is wave velocity and the Cauchy data are
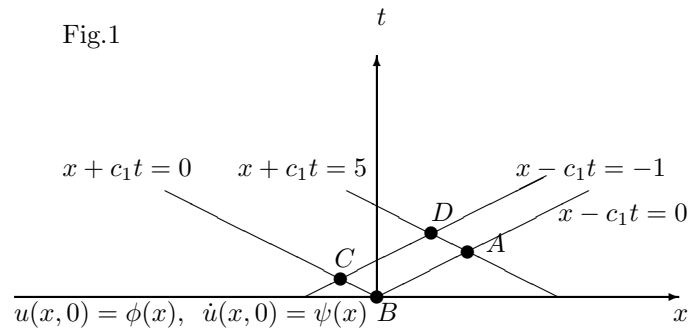
$$u(x,0) = \phi(x), \ \ \dot{u}(x,0) = \psi(x),$$

D'Alembert's solution :

$$u(x,t) = \frac{1}{2}\phi(x + c_1 t) + \frac{1}{2}\phi(x - c_1 t) + \frac{1}{2c_1}\int_{x-c_1 t}^{x+c_1 t} \psi(x)dx$$

2. Explain the paradox in the course Prove the diamond rule. (10 %)

$$u_A + u_C = u_B + u_D$$

3. Explain the paradox in the course using the results of **1** and **2**. (10 %)

Fig.1

$$x + c_1 t = 0 \qquad x + c_1 t = 5 \qquad\qquad x - c_1 t = -1$$

$$x - c_1 t = 0$$

$$u(x,0) = \phi(x), \ \ \dot{u}(x,0) = \psi(x)$$