工程數學 (四) - 偏微分方程作業七

Apr.30, 1998

I. Write down ten PDEs you have learned.

II. Design an experiment with initial displacement $\phi(x)$ from rest, such that at $t = t_0$, the velocity is zero everywhere. Also, Design an experiment with initial velocity $\psi(x)$ only, such that at $t = t_0$, the displacement is zero everywhere, i.e., find the constraint of $\phi(x)$ and $\psi(x)$. Do you have any comment for the limit in applying the diamond rule.

III. Prove the diamond rule for the case of only initial velocity.

IV. Reflection and transmission in two medium using diamond rule. Solve the PDE

$$u_{tt} = \begin{cases} c_1^2 u_{xx}, & for \;\; x > 0, \;\; t > 0, \quad medium \;\; density \; \rho_1 \\ c_2^2 u_{xx}, & for \;\; x < 0, \;\; t > 0, \quad medium \;\; density \; \rho_2 \end{cases}$$

with initial condition of displacement

$$u(x,0) = \begin{cases} f(x), & for \;\; x > 0 \\ 0, & for \;\; x < 0 \end{cases}$$

with initial condition of velocity

$$u_t(x,0) = \begin{cases} 0, & for \;\; x > 0 \\ 0, & for \;\; x < 0 \end{cases}$$

$u(x,t)$ is continuous across $x = 0$,

$$u(0^+, t) = u(0^-, t)$$

Force can be transmitted across $x = 0$,

$$\rho_1 c_1^2 u_x(0^+, t) = \rho_2 c_2^2 u_x(0^-, t)$$

(1). Determine the ratio of transmission and reflection.

V. Solve the PDE

$$u_{tt} = \begin{cases} 4u_{xx}, & for \;\; x < 0, \;\; t > 0 \\ 1u_{xx}, & for \;\; x > 0, \;\; t > 0 \end{cases}$$

with initial conditions

$$u(x,0) = 0, u_t(x,0) = 0$$

and $u(x,t)$ is continuous across $x = 0$, while

$$u_x(0^+, t) - u_x(0^-, t) = a \; sin(\omega t)$$

where a, ω are two constants.